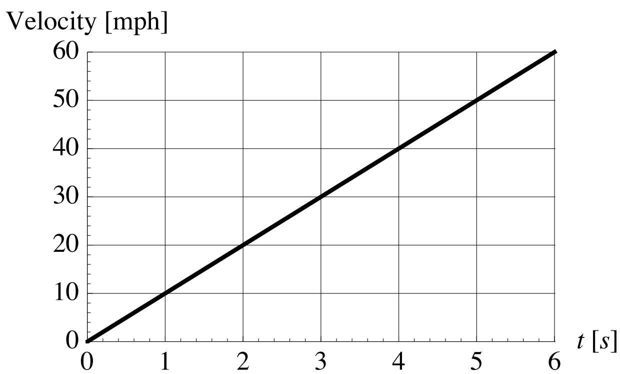
Estimating Instantaneous Velocity

Instantaneous velocity is the velocity of an object at any given instant in time. In a car, the speedometer gives instantaneous speed: if you accelerate onto a freeway from 0 to 60 mph, the speedometer tells you at any given instant what your speed is. As an aside, it's interesting to think about the fact that when you accelerate from 0 to 65 mph, you travel at all possible instantaneous speeds between 0 and 60 mph (which is an infinite number of instantaneous speeds!).

Here's a graph of what constant acceleration from 0 to 60 mph in 6 seconds would like:



The equation of the velocity as a function of time is linear:

$$v(t) = 10\,t$$

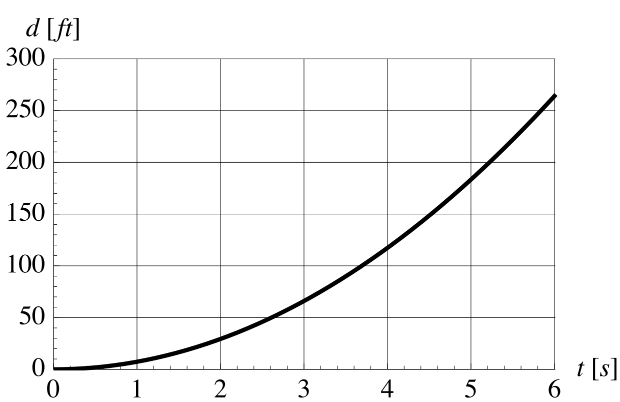
From this equation, we can get the instantaneous velocity at any instant in time. For example, at $t = 3.5$ s, the velocity is:

$$v(3.5) = 10 \cdot 3.5 = 35 \text{ mph}$$

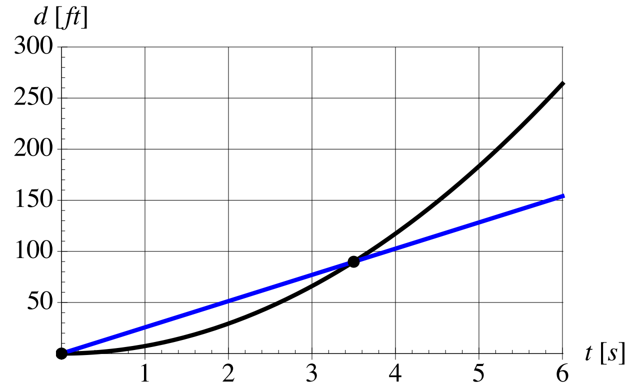
From your work in Physics, you may recall that if an object is undergoing constant acceleration from zero velocity (in this case 10 mph every second, which is 14.667 ft / s^2), then the position from the starting position is given by:

$$d(t) = \frac{1}{2}\,14.667\,t^2 = 7.333\,t^2 \text{ [feet]}$$

Here is a graph of the position versus time:



In previous notes, we reviewed how we could find the average velocity between any two points on a Position versus Time graph by finding the slope of the secant line that passes through those two points. For example, to find the average velocity from 0 to 3.5 seconds, let's first draw the corresponding secant line:



The slope of this secant line is, of course, the rise over the run:

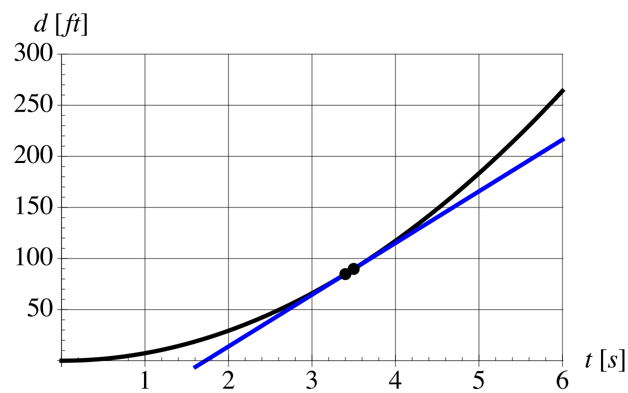
$$m = \frac{\Delta d}{\Delta t} = \frac{d(3.5) - d(0)}{3.5 - 0} = \frac{7.333\,(3.5)^2 - 0}{3.5 - 0} = 25.6667\ \text{ft}/s$$

So, the average velocity from 0 to 3.5 seconds was 25.67 feet per second.

From the velocity equation, we know that the instantaneous velocity at 3.5 s was 35 mph (or 51.333 ft/s). If we didn't have the velocity equation (or graph), how could we estimate the instantaneous velocity using the position equation (or graph)?

The answer is to approximate instantaneous velocity using average velocity.

To estimate the instantaneous velocity at 3.5 s we are going to choose another point in time that is really close to 3.5 s (but, of course, not equal to 3.5 s). Let's choose a time that's close: 3.4 s. Here's a graph of the secant line that goes through the position graph at 3.4 s and 3.5 s:



The slope of this secant line is:

$$m = \frac{\Delta d}{\Delta t} = \frac{d(3.5) - d(3.4)}{3.5 - 3.4} = \frac{7.333\,(3.5)^2 - 7.333\,(3.4)^2}{0.1} = 50.6\ \text{ft}/s$$

This is not far from the value obtained using the velocity equation: 51.333 ft/s.

Of course, we could make our answer more accurate if we chose a time even closer to 3.5, for example 3.49 s:

$$m = \frac{\Delta d}{\Delta t} = \frac{d(3.5) - d(3.49)}{3.5 - 3.49} = \frac{7.333\,(3.5)^2 - 7.333\,(3.49)^2}{0.01} = 51.26\ \text{ft}/s$$

That's not a bad estimate!