Evaluating Limits Algebraically: Type II

Type II Limits

Type II limits are of the form:

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} \quad \text{or} \quad \lim_{x \to -\infty} \frac{f(x)}{g(x)}$$

In other words, Type II limits are about the *End Behavior* of the function. An example of a Type II limit is:

$$\lim_{x \to \infty} \frac{2x^2 - x + 3}{3x^2 + x - 8}$$

To evaluate (find) the limit, we use the concept of *dominant terms*.

Dominant Terms

Let's start by looking at the numerator of the example limit: $2x^2 - x + 3$. This is obviously a quadratic equation so its graph would be a parabola. Rather than look at the graph, let's take a close look at how the value of the expression changes as x gets larger and larger, which is summarized in this table:

x	**$2x^2$**	**−**	**x**	**+**	**3**	**=**	**Result**
0	0	−	0	+	3	=	3
1	2	−	1	+	3	=	4
2	8	−	2	+	3	=	9
10	200	−	10	+	3	=	193
100	20 000	−	100	+	3	=	19 903
1000	2 000 000	−	1000	+	3	=	1 999 003
10 000	200 000 000	−	10 000	+	3	=	199 990 003

In this table there is a pattern, as x grows (gets larger and larger) the $2x^2$ term "dominates" and the value of the full expression is approximately the value of the $2x^2$ term:

$$2x^2 - x + 3 \approx 2x^2$$

The larger x gets, the better the approximation. While we won't prove it here, it can be shown that as x goes towards infinity, the value of $2x^2 - x + 3$ approaches $2x^2$. In other words, using limit notation:

$$\lim_{x \to \infty} 2x^2 - x + 3 = \lim_{x \to \infty} 2x^2$$

Finding Type II Limits

Using this concept of dominant terms, it is possible to find limits such as

$$\lim_{x \to \infty} \frac{2x^2 - x + 3}{3x^2 + x - 8}$$

We start be simplifying both the numerator and the denominator so that they contain only the dominant term:

$$\lim_{x \to \infty} \frac{2x^2 - x + 3}{3x^2 + x - 8} = \lim_{x \to \infty} \frac{2x^2}{3x^2}$$

We can then simplify the expression by getting rid of common factors in the numerator and the denominator:

$$\lim_{x \to \infty} \frac{2x^2 - x + 3}{3x^2 + x - 8} = \lim_{x \to \infty} \frac{2x^2}{3x^2} = \lim_{x \to \infty} \frac{2}{3} = \frac{2}{3}$$