

The Rules of Dominant Terms

The *Rules of Dominant Terms* are used to determine which term in an expression is going to race towards infinity the fastest, because the term that does is going to dominate the expression.

Rule 1

Rule 1 can be summarized as

$$\text{logarithm} < \text{power} < \text{exponential}$$

This rule is saying

- a. a term that is a power expression (x^2 , $x^{1/2}$, x^{100} , ...) will dominate a logarithmic term ($\log(x)$, $\log_2(x)$, ...).
- b. a term that is an exponential expression (2^x , e^x , 3^{10x} , ...) will dominate both a power term and a logarithmic term.

For example, let's find

$$\lim_{x \rightarrow \infty} \frac{x^4}{2^x}$$

Since the 2^x dominates, we can simplify the numerator to any expression that 2^x dominates, including the simplest of expressions, 1:

$$\lim_{x \rightarrow \infty} \frac{x^4}{2^x} = \lim_{x \rightarrow \infty} \frac{1}{2^x} = 0$$

Rule 2

Rule 2 explains how exponential terms dominate each other; for example:

$$1.1^x < 2^x < 8^x$$

In words, this rule is saying that the exponential term with the largest base dominates.

Rule 3

Rule 3 explains how power terms dominate each other; for example:

$$\sqrt{x} < x^2 < x^{10}$$

In words, this rule is saying that the power term with the largest exponent dominates.

Rule 4

Rule 4 explains how logarithmic terms dominate each other; for example:

$$\log_{10}(x) < \log_5(x) < \log_2(x)$$

In words, this rule is saying that the log term with the smallest base dominates.