Summation Notation

When representing the approximate area of the area under curve as the sum of the areas of trapezoids or rectangles, it can get quite tedious to have to express the area as a long list of numbers added together. To avoid this, mathematicians have developed a short-hand way of representing long sums: *summation notation* (which is also called *sigma notation*).

Here is an example of summation notation and it corresponding "long-form" expression:

$$\sum_{k=1}^{3} k^2 = 1^2 + 2^2 + 3^2$$

The capital Greek letter sigma, Σ, is used to represent the sum. The letter k is called the *index* of the sum and, in this example the index starts at 1 (the *starting value*) and goes up to 3 (the *ending value*) in increments of 1. In other words, the values of the increments are consecutive integer values only. In this example, k takes on the values 1, 2, and 3. To form the sum, we take the expression k^2 (which is called the *argument* of the sum), substitute our k values and add them all together.

Example 1

Expand the sum

$$\sum_{j=1}^{10} j$$

Solution

$$\sum_{j=1}^{10} j = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

Example 2

Expand the sum

$$\sum_{n=1}^{4} n(3n - 1)^2$$

Solution

$$\sum_{n=1}^{4} n(3n - 1)^2 = 1\,(3 - 1)^2 + 2\,(3 \cdot 2 - 1)^2 + 3\,(3 \cdot 3 - 1)^2 + 4\,(3 \cdot 4 - 1)^2$$