The Intuitive Definition of a Limit

In our work with functions, we have been looking at how a function behaves

1. at a point, for example, the value of a function for a given input, $f(a)$.
2. as it approaches a vertical asymptote, for example, as $x \to 0^+$, $y \to \infty$.
3. as $x \to \infty$ and as $x \to -\infty$ (end behavior), for example, $y \to 3$ or $y \to -\infty$.

Calculus introduces a new idea: not just the value of a function at a point a, but what $f(x)$ looks like very, very close to a. In fact, we may be interested in these values at nearby points x when f isn't defined at the point a!

When you graph a function $y = f(x)$, most of the time you can guess what the value of the function is at some point, say, $f(3)$, by knowing the values of $f(x)$ when x is very close to 3. One way to think about this is to assume you have the graph $y = f(x)$ for $2 < x < 4$, except at $x = 3$. Can you make a reasonable accurate guess as to the value of $f(3)$? If so, and this value is L, we say that the limit of $f(x)$ exists at $x = 3$ and use the notation:

$$\lim_{x\to 3} f(x) = L$$

Example

Consider the following table of function values:

x	$g(x)$
2.999	3.997
3	
3.0001	4.005
3.01	4.02

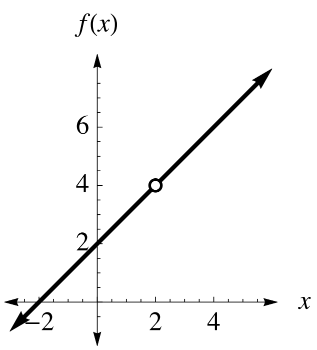
It is reasonable to guess that $g(3) = 4$, so:

$$\lim_{x\to 3} g(x) = 4$$

An Important Note

It is important to note that $\lim_{x\to 3} f(x)$ does not need to equal $f(a)$! Here is a simple example...

Below is a graph of the rational function $f(x) = \frac{x^2-4}{x-2} = \frac{(x+2)(x-2)}{x-2}$,



This function has a hole at $x = 2$, so the function does not exist at that point ($x = 2$ is not in the domain of the function). However,

$$\lim_{x\to 2} f(x) = 4$$