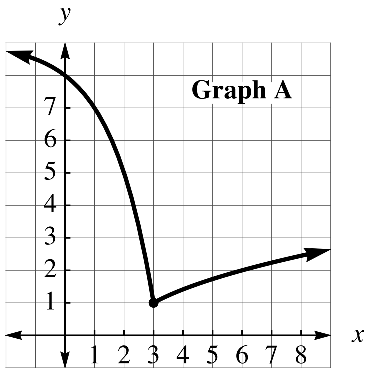
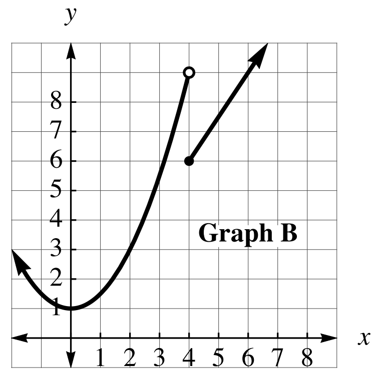
Continuity (informal)

We say that a function is **continuous** if the graph of the function can be drawn without lifting your pencil from the paper. Graph A (below, left) is an example of a piecewise function that is continuous:





Its equation is:

$$f(x) = \begin{cases} 9 - 2^x & \text{for } x \le 3 \\ \sqrt{x-2} & \text{for } x > 3 \end{cases}$$

By looking at the graph we can see that it is continuous because it can be drawn with lifting a pencil from the paper. We also can tell from its equation that it is continuous because at $x = 3$ (the location of the fence), the two expressions are equal:

$$9 - 2^3 = \sqrt{3-2}$$
$$1 = 1$$

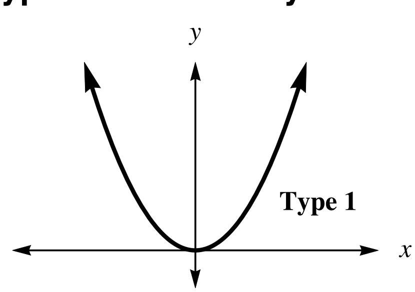
When the pieces of a function are not connected, we say the function is **not continuous**. Graph B (above, right) shows an example of a function that is not continuous. It's equation is:

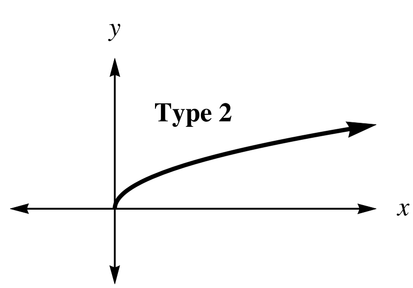
$$g(x) = \begin{cases} 0.5\,x^2 + 1 & \text{for } x < 4 \\ 1.5\,x & \text{for } x \ge 4 \end{cases}$$

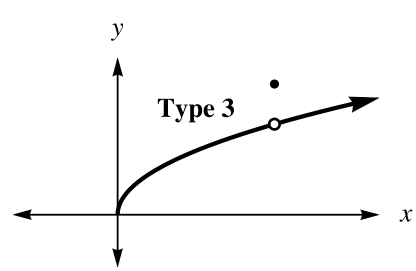
We can also tell from its equation that it is not continuous because at $x = 4$:

$$0.5\,(4)^2 + 1 \ne 1.5\,(4)$$
$$9 \ne 6$$

Types of Continuity







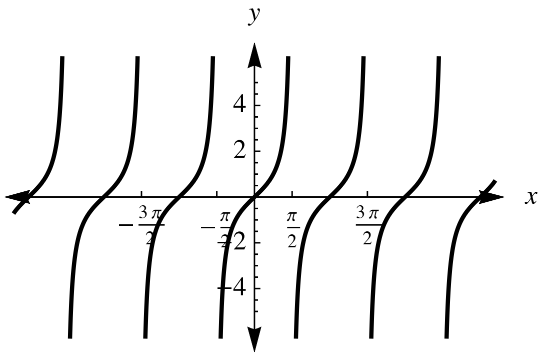
The graphs above show examples of three possible types of continuity a function can have:

1. Continuous for all real values of x. For example, $y = x^2$:
2. Continuous on its domain only. For example, $y = \sqrt{x}$:
3. Discontinuous on its domain. For example,

$$f(x) = \begin{cases} \sqrt{x} & \text{for } x \neq 3 \\ 4 & \text{for } x = 3 \end{cases}$$

A Note on "Discontinuous on its Domain" (Challenge)

If all the pieces of a function are not connected (so that it cannot be drawn without lifting the pencil from the paper) but those points of disconnection are not in the domain of the function, the function is still considered to be continuous (so long as each piece is by itself continuous). An example of this is the Tangent function:



The tan function is undefined at certain points (such as $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2},$ and $\frac{3\pi}{2}$), so these points are not in its domain. Even though this function cannot be drawn without lifting the pencil from the paper, each piece in its domain can, so tan(x) is a continuous function.