Sketching Rational Functions, Part 2

In the last set of notes, we sketched the graph of a rational function that had a vertical asymptote and a slant asymptote. Now let's graph a rational function that ends up having a hole (a point of discontinuity). Let's illustrate with an example:

Example

Graph the rational function:

$$f(x) = \frac{x^2 + 2x - 8}{x - 2}$$

Step 1: Find the *x* values that are not in the domain of the function

Since we cannot divide by zero, the x values that are not in the domain of the function are the ones that make the denominator zero:

$$x - 2 = 0$$
$$x = 2$$

So, at $x = 1$, there is either a hole or a vertical asymptote.

Step 2: Determine where there are holes and where there are vertical asymptotes

Using the x values obtained in Step 1, we evaluate the function at those values to see if the function is Undefined at that value (and hence a vertical asymptote) or Indeterminate (and hence a hole). For our example:

$$f(-1) = \frac{(2)^2 + 2(2) - 8}{(2) - 2} = \frac{4 + 4 - 8}{0} = \frac{0}{0} = \text{Indeterminate}$$

Since the function is indeterminate at $x = 2$ it could have a hole at $x = 2$.

Step 3: Simplify the function

To see if there is a hole, we have to simplify the function. We simplify the function using Wolfram Alpha (or a similar app), factoring (Honors) or polynomial division (Honors).

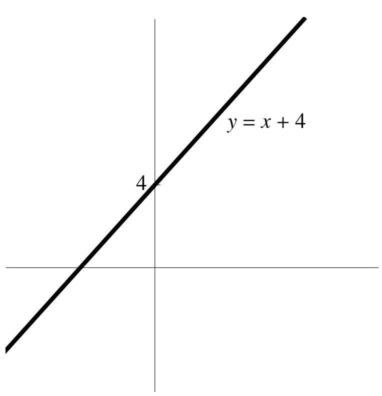
Using Wolfram Alpha, we can see that

$$f(x) = \frac{x^2 + 2x - 8}{x - 2} = \frac{(x - 2)(x + 4)}{x - 2} = x + 4$$

Since the functional simplifies to a polynomial, there is a hole (and not an asymptote) at $x = 2$.

Step 4: Graph the simplified function

Draw a simple sketch of the function, labeling significant points, such as the y-intercept. Label the graph with the function rather than try to draw it precisely.



Step 5: Add the hole to the graph

Use the simplified function to find the y value for the hole:

$$f(x) = x + 4$$
$$f(2) = (2) + 4 = 6$$

Plot this hole on the graph.