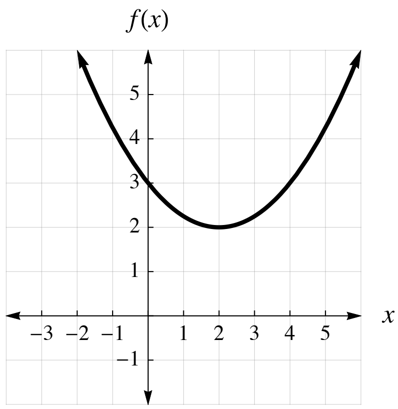
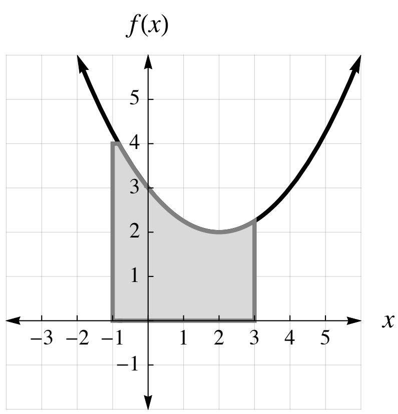
Approximating the Area under a Curve with Rectangles

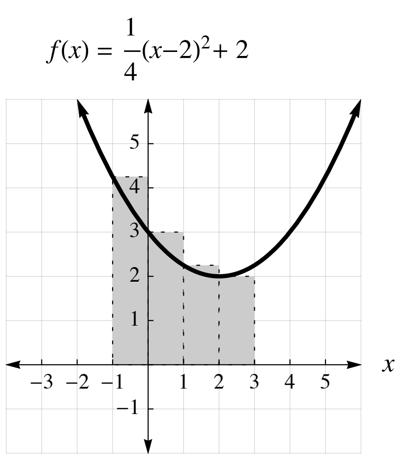
Let's graph the quadratic equation $f(x) = \frac{1}{4}(x-2)^2 + 2$



Now, let's assume we want to find the area between this parabola and the x-axis, when x is greater than -1 and less than 3; that is: $-1 \le x \le 3$. That area is shaded below.

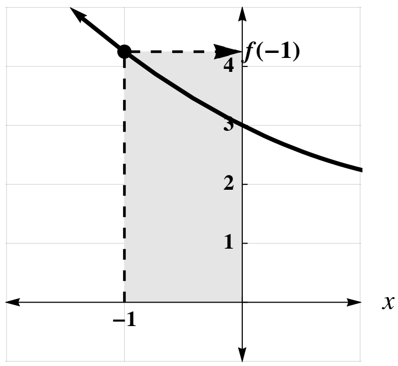


To approximate this area, we can take it apart into four rectangles, as shown below. Obviously there will be some error since the rectangles don't perfectly match the shaded area above.



For this example, all the widths of the rectangles are are the same and equal to 1 unit. The lengths of the rectangles (which could also be thought of as their heights), can be obtained by noticing that the the upper left corners of each rectangle are on the parabola, so the function $f(x) = \frac{1}{4}(x-2)^2 + 2$ can give us the rectangle heights. Here's a diagram showing this in detail:

$$f(x) = \frac{1}{4}(x-2)^2 + 2$$



Since

$$f(-1) = \frac{1}{4}(-1-2)^2 + 2 = \frac{1}{4}(-3)^2 + 2 = \frac{9}{4} + 2 = \frac{9}{4} + \frac{8}{4} = \frac{17}{4}$$

the area of that first rectangle is simply:

$$A = l \cdot w = \left(\frac{17}{4}\right)(1) = \frac{17}{4} = 4.25$$

Now that we have a way of finding the area for each rectangle, we can create a table to “stay organized” and find the total area of all the rectangles:

Rectangle	x	$f(x) = \frac{1}{4}(x-2)^2 + 2$	$A = l \cdot w$
1	-1	$f(-1) = \frac{1}{4}(-1-2)^2 + 2 = \frac{17}{4}$	$\frac{17}{4}$
2	0	$f(0) = \frac{1}{4}(0-2)^2 + 2 = 3$	3
3	1	$f(1) = \frac{1}{4}(1-2)^2 + 2 = \frac{9}{4}$	$\frac{9}{4}$
4	2	$f(2) = \frac{1}{4}(2-2)^2 + 2 = 2$	2

Adding up the area of all 4 rectangles gives us

$$A_{\text{total}} = \frac{17}{4} + 3 + \frac{9}{4} + 2 = \frac{23}{2} = 11.5 \text{ square units.}$$