Finding Inverse Functions

Let's say we have a simple linear function

$$f(x) = 2x - 5$$

and we want to find its inverse; in other words we want to find a new function, $g(x)$, that will take the expression

$$2x - 5$$

and do some mathematical operations on it so that the expression gets reduced to x. The first obvious step is to add 5, leaving us with

$$2x$$

If we then divide by 2, we have

$$x$$

So, if we have our function g take its input and add 5 from it and then divide the result by 2, it will be the inverse of f:

$$g(x) = \frac{x+5}{2}$$

We can double-check this:

$$g(f(x)) = g(2x+5) = \frac{(2x+5)-5}{2} = \frac{2x}{2} = x$$

Note how the steps we take are exactly the same steps if we want to get x by itself:

$$f(x) = 2x - 5$$
$$f(x) + 5 = 2x$$
$$\frac{f(x)+5}{2} = x$$

This last equations is saying that if we take the output of f, which is $f(x)$, and add 5 to it and then divide by two, we will get x. This is $g(x)$!

$$g(x) = \frac{x+5}{2}$$

Example Problem

Find the inverse of $f(x) = 2x^2 - 3$.

Solution

Rearrange to get x by itself:

$$f(x) = 2x^2 - 3$$
$$f(x) + 3 = 2x^2$$
$$\frac{f(x)+3}{2} = x^2$$
$$\sqrt{\frac{f(x)+3}{2}} = x$$

So our inverse function is:

$$g(x) = \sqrt{\frac{x+3}{2}}$$

Example Problem (Challenge)

Find the inverse of the function $f(x) = 5 - \frac{1}{2x}$.

Solution

Rearrange to get x by itself:

$$f(x) = 5 - \frac{1}{2x}$$

$$f(x) - 5 = \frac{-1}{2x}$$

$$-2(f(x) - 5) \cdot x = \frac{1}{x} \cdot x$$

$$-2x(f(x) - 5) = 1$$

$$x = \frac{1}{-2(f(x) - 5)}$$

So our inverse function is:

$$g(x) = \frac{-1}{2(x - 5)}$$