

## Polynomial Functions

A polynomial function of degree  $n$  (where  $n$  is natural number) has the following form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

where the  $a$ 's are real numbers and are referred to as the coefficients of the term. Of course, any of the coefficients could be zero. Examples of polynomial functions include:

$$f(x) = x^2 - 2x + 1$$

$$f(x) = 7x^8 - 6x^3$$

$$f(x) = x^{10}$$

The *degree* of a polynomial is the highest power of  $x$  in the formula. For example, the powers of the above polynomials are 2, 8 and 10, respectively. Some polynomials with special degrees are given specific names. The table below summarizes some of these:

Polynomial Form	Degree	Example	Name
$f(x) = a_0$	0	$f(x) = 4$	Constant Function
$f(x) = a_1 x + a_0$	1	$f(x) = -3x + 2$	Linear Function
$f(x) = a_2 x^2 + a_1 x + a_0$	2	$f(x) = 5x^2 - 4x + 2$	Quadratic Function
$f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$	3	$f(x) = 2x^3 + 2$	Cubic Function
$f(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$	4	$f(x) = x^4 - 9x^3 + 2$	Quartic Function

Functions that contain other operations, such as square roots, are not polynomial functions:

$$f(x) = x^2 - 1/x$$

$$f(x) = x^3 - x^2 - \sqrt{x}$$

$$f(x) = x^4 + 3^x$$

The term “polynomial” comes from “poly” meaning “many” and “nomial” meaning “term”; thus, a polynomial is an expression with “many terms”. A *monomial* is polynomial with only one term, such as

$$f(x) = x^2 \quad f(x) = 5 \quad f(x) = -4x^{12} \quad f(x) = ax^2$$

A *binomial* is a polynomial with two terms (two monomials), such as

$$f(x) = 5x^2 - 4 \quad f(x) = 2x - 6 \quad f(x) = x^3 - x^2 \quad f(x) = ax^2 + bx$$

A *trinomial* is a polynomial with three terms (three monomials), such as

$$f(x) = 5x^6 - 3x^3 - 4 \quad f(x) = rx^3 + sx - 5 \quad f(x) = ax^2 + bx + c$$