Rational Functions

Rational functions are of the form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$. The following are all examples of rational functions:

$$f(x) = \frac{1}{x}$$

$$f(x) = \frac{1}{3x - 1}\left(2x^2 - 4\right)$$

$$f(x) = \frac{a\,x + b}{c\,x + d}$$

Since $q(x) \neq 0$, the domain of a rational function is often limited. For example, the following rational function has the set of all real numbers except $x = 1$ and $x = -1$.

$$f(x) = \left(3x^3 - 3x - 3\right) \Big/ \left(x^2 - 1\right)$$

Note that $q(x) = 1$ is a perfectly valid function. That means that all polynomial functions (including constant, linear, quadratic and cubic functions) are rational functions.