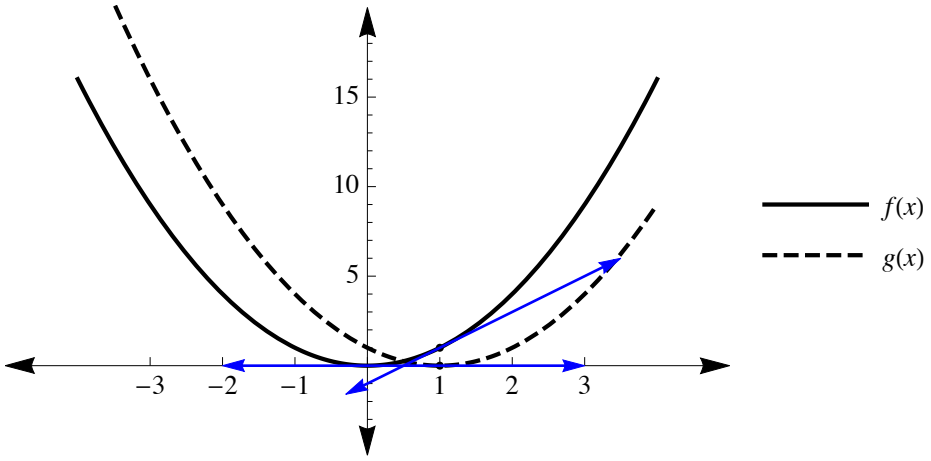
Slope Function for f(x) = (x - h)^n (Horizontal Shift)

Let's compare and contrast the graphs $f(x) = x^2$ and $g(x) = (x-1)^2$.



For a function $f(x)$, subtracting a positive number, h, from the independent variable, $f(x-h)$, causes the function to translate (shift) to the right by h units. Adding a positive number, h, to the independent variable, $f(x+h)$, causes the function to translate (shift) to the left by h units.

Clearly, at $x = 1$, $g(x)$ has zero slope which is the slope that $f(x)$ has at $x = 0$. By drawing a series of tangent lines on both curves and finding their slopes, we can see the relationship:

x	−4	−3	−2	−1	0	1	2	3	4
$f'(x)$	−8	−6	−4	−2	0	2	4	6	8
$g'(x)$	−10	−8	−6	−4	−2	0	2	4	6

For any given x value the slope of $g(x)$ is equal to the slope of $f(x-1)$. Since $f'(x) = 2x$,

$$g'(x) = f'(x-1) = 2(x-1)$$

In general, if $f(x) = (x-h)^n$, where h is some real-number, then

$$f'(x) = n(x-h)^{n-1}$$

Example

If $f(x) = (x-5)^8$ what is $f'(x)$?

Solution

$$f'(x) = 8(x-5)^{8-1} = 8(x-5)^7$$