Homework #12

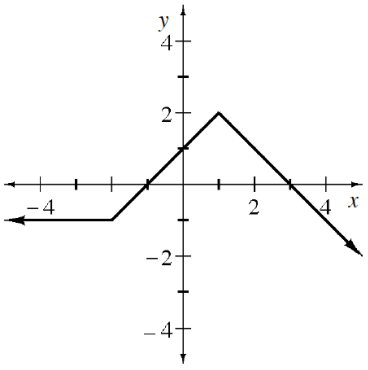
First & Last Name: _____ **Class:** _____

For homework to be graded, it must be *fully completed*. This means you must **show your work**.

1. **[Challenge]** Determine the exact value(s) of x in the domain $0 \le x \le 2\pi$ if:
   - **a.** $\sin(x) = -\frac{1}{2}$, $\tan(x) > 0$
   - **b.** $\cot(x)$ is undefined, $\cos(x) > 0$
   - **c.** $\csc(x) = \sqrt{2}$, $\sin(x) > \cos(x)$

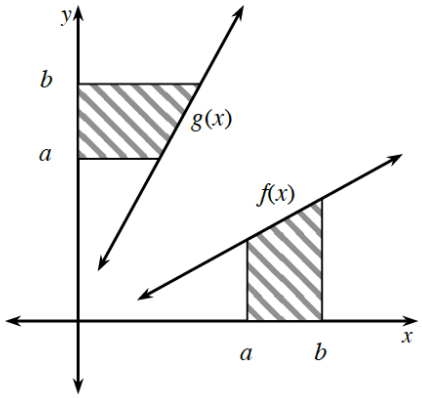
2. Given $f(x) = 2x^2 - 3$:
   - **a.** Evaluate $f(2)$.
   - **b.** Without writing the equation of the inverse, determine $f^{-1}(5)$. Explain your process.
   - **c.** **[Challenge]** Solve for x if $f(x+2) - f(x-2) = 64$.

3. Using the graph of $y = f(x)$ below sketch the following transformations. Desmos (desmos.com/calculator/zg58w7musa).



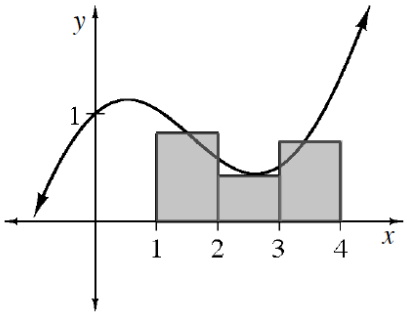
   - **a.** $y = -f(x)$
   - **b.** $y = f(x+3)$
   - **c.** $y = f(x) - 2$
   - **d.** **[Challenge]** $y = |f(x)|$

4. **[Challenge]** Sandra is playing around with inverses and thinks she has discovered something interesting. She thinks that if $f(x) = g^{-1}(x)$, then the areas of the regions shaded below are equal. Use $f(x) = \frac{1}{3}x+1$ with $a = 3$ and $b = 5$ to verify Sandra's conjecture.



5. To estimate the area under a curve, rectangles are often the easiest shape to use. However, there are different ways to choose the heights of the rectangles. You have already used left endpoint and right endpoint rectangles. Another way is to use midpoint rectangles, which have heights defined at the midpoints of the intervals. For example, for the function $f(x) = \frac{1}{2}x + \cos(x)$ graphed below, the first midpoint rectangle has a height of $f(1.5) \approx 0.821$.

Calculate the height of the other two rectangles then use them to approximate the area under the curve for $1 \le x \le 4$.

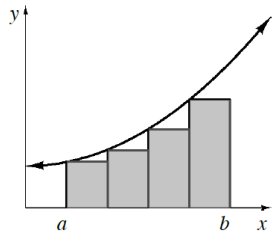


6. Which is Better? Part One

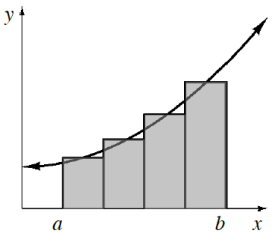
Below are different sets of rectangles to approximate the area under a curve for the same interval. Look at the three different sets of rectangles and decide which will best approximate the area under the curve of this function for $a \le x \le b$.

a. Explain why your choice will determine the best approximation for the area.

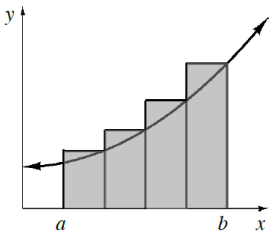
b. Will left endpoint rectangles always be an underestimate for any function? Explain.



Left Endpoint Rectangles



Midpoint Rectangles



Right Endpoint Rectangles