

Homework #25

First & Last Name: _____ **Class:** _____

For homework to be graded, it must be *fully completed*. This means you must **show your work**.

1. A helium balloon is released from the ground and floats upward. The height of the balloon is shown at the following times:

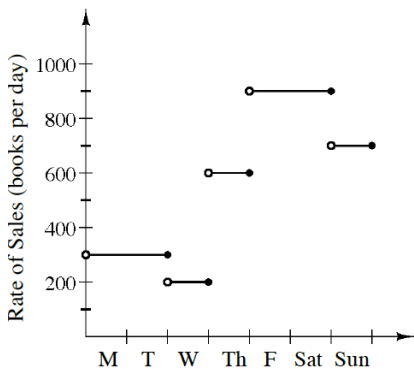
Time [s]	0	1	2	3	4	5	6	7	8	9	10
Height [feet]	0	50	98	144	188	230	270	308	344	378	410

- What is the average velocity over the first 10 seconds of the balloon's flight? Over the first 5 seconds?
 - Calculate the finite differences for the heights. How is the velocity changing? Explore this using [Desmos](https://www.desmos.com/calculator/kjguaihhjs) (desmos.com/calculator/kjguaihhjs).
 - What do the finite differences tell you about the height function for the balloon?
2. Examine the expanded sums below and write the equivalent sigma notation.
- $\frac{2}{3}f(-2 + \frac{2}{3} \cdot 0) + \frac{2}{3}f(-2 + \frac{2}{3} \cdot 1) + \frac{2}{3}f(-2 + \frac{2}{3} \cdot 2) + \frac{2}{3}f(-2 + \frac{2}{3} \cdot 3)$
 - $\frac{1}{2}f(6 + \frac{1}{2} \cdot 0) + \frac{1}{2}f(6 + \frac{1}{2} \cdot 1) + \frac{1}{2}f(6 + \frac{1}{2} \cdot 2) + \frac{1}{2}f(6 + \frac{1}{2} \cdot 3) + \frac{1}{2}f(6 + \frac{1}{2} \cdot 4)$
3. The Intermediate Value Theorem is sometimes used to prove that roots exist. For example, $f(x) = 5\sqrt[3]{x-2}$ is a continuous function. Given $f(2) = -4$ and $f(3) = 1$, does f have a root somewhere between $x = 2$ and $x = 3$? Why or why not? [Desmos](https://www.desmos.com/calculator/ht473ho5qm) (desmos.com/calculator/ht473ho5qm).
4. Write a Riemann sum for a general function f to estimate the area under the curve for $2 \leq x \leq 5$ using n left endpoint rectangles of equal width if:
- $n = 3$ rectangles
 - $n = 9$ rectangles
 - $n = 300$ rectangles
5. Jamal wrote the following Riemann sum to estimate the area under $f(x) = 3x^2 - 2$.

$$\sum_{i=0}^9 \frac{1}{2} f\left(-3 + \frac{1}{2}i\right)$$

[Desmos](https://www.desmos.com/calculator/xz3qddhiye) (desmos.com/calculator/xz3qddhiye).

- Draw a sketch of the region. How many rectangles did he use?
 - For what domain of f did Jamal estimate the area?
 - Use the summation feature of Desmos to approximate the area using Jamal's Riemann sum.
6. The manager of Books-To-Go knows that the rate of daily sales (in books per day) varies over the course of a week. This rate can be represented by the step function shown in the graph below. Using this data, calculate how many books this store sold during this week. What is the average number of books sold per day?



7. For each description below, write a limit equation and sketch a possible function.

- a. As $x \rightarrow 0$, $y \rightarrow 9$.
- b. As x gets closer to 3 on both sides, $f(x)$ becomes increasingly large.
- c. Which of the limits from parts (a) and (b) exist? Explain your reasoning.

8. [Challenge] Given

$$f(x) = \begin{cases} x^2 & \text{for } x < 1, \text{ but } x \neq 1 \\ 3 & \text{for } x = 1 \\ 2x - 1 & \text{for } 1 < x < 3 \\ 4 & \text{for } x \geq 3 \end{cases}$$

[Desmos](https://www.desmos.com/calculator/0nppaejnel) (desmos.com/calculator/0nppaejnel).

- a. Sketch $y = f(x)$.
- b. For what values of x is f not continuous?