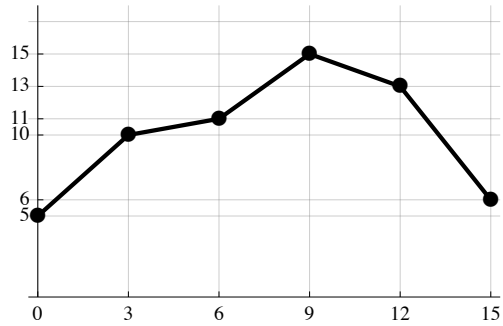
The Trapezoid Rule

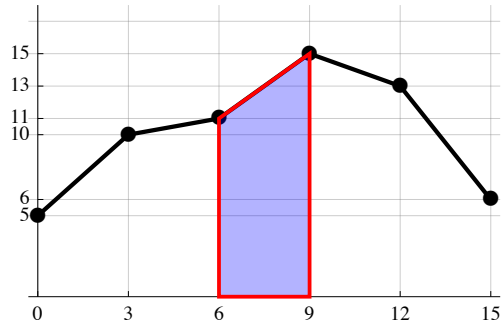
Below is a table of data:

x	0	3	6	9	12	15
$f(x)$	5	10	11	15	13	6

If we graph this data (a *scatterplot*) and "connect" the scatter plot, we end up with a a continuous function as shown below:



We can use five trapezoids to approximate the area under f on $0 \le x \le 15$. One trapezoid is shown below as an example:



Since the area of a trapezoid is $A = \frac{1}{2}(a+b)\,h$, the area of the all five trapezoids is:

$$\text{Area} = \frac{1}{2}(5+10)\cdot 3 + \frac{1}{2}(10+11)\cdot 3 + \frac{1}{2}(11+15)\cdot 3 + \frac{1}{2}(15+13)\cdot 3 + \frac{1}{2}(13+6)\cdot 3$$

Since the width of each trapezoid is 3 units, we can re-write this as:

$$\text{Area} = \frac{3}{2}(5+10) + \frac{3}{2}(10+11) + \frac{3}{2}(11+15) + \frac{3}{2}(15+13) + \frac{3}{2}(13+6)$$

And then we can factor out the $\frac{3}{2}$:

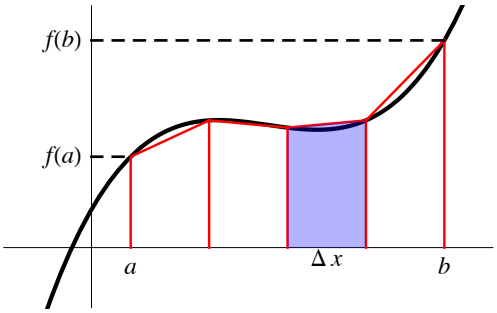
$$\text{Area} = \frac{3}{2}(5+10+10+11+11+15+15+13+13+6) = \frac{3}{2}(5+2\cdot 10+2\cdot 11+2\cdot 15+2\cdot 13+6)$$

Notice this pattern in the above equation: for the first trapezoid, its left side length (5) is only used once and for the right trapezoid, its right side length (6) is also only used once. For all the other trapezoids, their side lengths are used twice. This pattern is the basis for a rule for approximating area under a curve: The Trapezoid Rule.

Generalization

The *Trapezoid Rule* is a formula for approximating the area under the curve of any function, $f(x)$, between $x = a$ and

$x = b$ if the widths, Δx, of all the trapezoids are of equal size. This is illustrated in the graph below (which is showing the desired area as only four trapezoids).



The general formula is:

$$A = \frac{\Delta x}{2}\left(f(a) + 2\,f(a + \Delta x) \;+\; 2\,f(a + 2\,\Delta x) \;+ 2\,f(a + 3\,\Delta x) + \;\cdots + 2\,f(b - \Delta x) + f(b)\right)$$