Evaluating Limits Algebraically: Type I

Rather than using Desmos to graph functions to find limits, we can evaluate (find) limits by using algebra. How we do this depends on the type of limit we're being asked to evaluate.

Type I

Type I limits are of the form:

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

where a is some real number and $f(x)$ and $g(x)$ are functions. For example:

$$\lim_{x \to 2} \frac{3\,x^3}{4^x}$$

Evaluating these types of limits involves 1 or, possibly 2 steps.

Step 1

The step for this type of limit is to evaluate

$$\frac{f(a)}{g(a)}$$

In other words, we need to plug in the a value for the expressions in the numerator and denominator. For our example:

$$\frac{f(a)}{g(a)} = \frac{f(2)}{g(2)} = \frac{3\,(2)^3}{4^2} = \frac{3 \cdot 8}{16} = \frac{3}{2} = 1.5$$

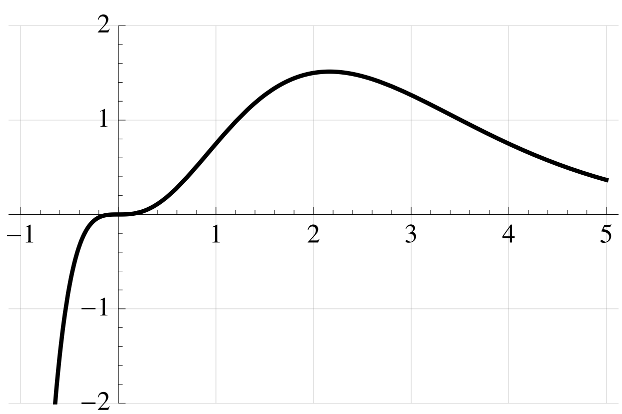
If $\frac{f(a)}{g(a)}$ is a real number, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}$$

So, for our example:

$$\lim_{x \to 2} \frac{3\,x^3}{4^x} = \frac{f(2)}{g(2)} = \frac{3}{2}$$

Here's the corresponding graph to help visualize:



If $\frac{f(a)}{g(a)}$ is a **not** real number, then we have to go to Step 2...

Step 2

If $\frac{f(a)}{g(a)}$ is not a real number, for example if

$$\frac{f(a)}{g(a)} = \frac{0}{0} \text{ or } \infty \text{ or } -\infty \text{ or } \frac{\infty}{\infty} \text{ or } \frac{-\infty}{\infty} \text{ or } \frac{\infty}{-\infty} \text{ or } \frac{b}{0} \text{ where } b \text{ is some real number}$$

then we must try to simplify $\frac{f(x)}{g(x)}$.

For example, consider the limit:

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$$

If we perform Step 1, we get

$$\frac{f(a)}{g(a)} = \frac{f(2)}{g(2)} = \frac{4 + 2 - 6}{2 - 2} = \frac{0}{0}$$

This means that to try and find the limit algebraically, we must try to simplify the expression $\frac{f(x)}{g(x)}$. There are a couple ways to do this, including:

1. Using the Wolfram Alpha website:

   Simplify $(x \wedge 2 + x - 6) / (x - 2)$

2. Factoring:

$$\frac{x^2 + x - 6}{x - 2} = \frac{(x+3)(x-2)}{(x-2)} = x + 3$$

Regardless of the method we use to simplify the expression, once we have simplified $\frac{f(x)}{g(x)}$, we then find the limit for that simplified expression (using Step 1):

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \to 2} (x + 3) = 2 + 3 = 5.$$

Here's the corresponding graph to help visualize: