Factoring Review (Honors)

Distribution of Multiplication over Addition

In the past, you learned how to multiple two binomial terms to obtain a quadratic expression. One way of doing this, is to multiply all pairs of terms:

$$(x+3)\,(x-1) = x^2 - x + 3\,x - 3 = x^2 + 2\,x - 3$$

Another way of thinking about this multiplication is an *area diagram*:

	x	$+\;3$
x	x^2	$3\,x$
$+\;-1$	$-x$	-3

The area of the whole rectangle must equal the sum of the four smaller rectangles:

$$(x+3)\,(x-1) = x^2 + 3\,x - x - 3 = x^2 + 2\,x - 3$$

You probably also learned how to extend these methods to multiply more complicated expressions:

$$\begin{aligned}(x+5)\left(x^2 - 2\,x + 4\right) = {} & x^3 - 2\,x^2 + 4\,x \\ & \qquad\; 5\,x^2 - 10\,x + 20 \\ \hline & x^3 + 3\,x^2 - 6\,x + 20\end{aligned}$$

This can also be multiplied with an area diagram:

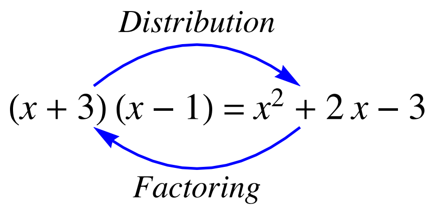
	x^2	$+\;-2\,x$	$+\;4$
x	x^3	$-2\,x^2$	$4\,x$
$+\;5$	$5\,x^2$	$-10\,x$	20

The area of the whole rectangle must equal the sum of the six smaller rectangles:

$$(x+5)\left(x^2 - 2\,x + 4\right) = x^3 + 3\,x^2 - 6\,x + 20$$

Factoring

Factoring is the *inverse* of Distribution of Multiplication over Addition; that is, factoring undoes distribution:



Looking for Patterns

To understand how to factor, it is helpful to see the patterns that emerge when we multiply binomials. The following table shows a number of examples for multiplications of the form:

$$(x+m)(x+n) \;=\; x^2+b\,x+c$$

Here’s the table:

$(x+m)(x+n) = x^2+b\,x+c$	m	n	b	c
$(x+1)(x+2) = x^2+3\,x+2$	1	2	3	2
$(x-1)(x+2) = x^2+x-2$	−1	2	1	−2
$(x+2)(x+3) = x^2+5\,x+6$	2	3	5	6
$(x+2)(x-3) = x^2-x-6$	2	−3	−1	−6
$(x-3)(x+3) = x^2-9$	−3	3	0	−9
$(x+4)(x-1) = x^2+3\,x-4$	4	−1	3	−4
$(x+5)(x+5) = x^2+10\,x+25$	5	5	10	25

From this table, it should be easy to see that:

1. $m+n = b$
2. $m \cdot n = c$

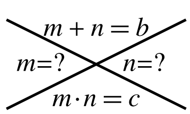
This can also be proven algebraically by distributing:

$$(x+m)(x+n) = x^2+m\,x\;+n\,x+m\,n = x^2+(m+n)\,x+m\,n$$

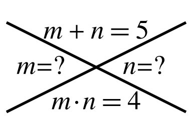
When we are factoring, we are given b and c (since they are part of the expression) and we are looking to find two number, m and n, such that $m+n = b$ and $m \cdot n = c$.

The “X” Rule for $a = 1$

This method for factoring can be summarized graphically:



Here is an example for $x^2+5\,x+4$



The *X-rule* is essentially asking: What two numbers when added together equal 5 and when multiplied together equal 4? The answer is 1 and 4, so:

$x^2 + 5x + 4 = (x+1)(x+4)$

The "X" Rule for $a \neq 1$

When the a coefficient of the quadratic is not equal to 1, factoring can be a bit more challenging. There is, however, a modified *X-rule* that can be used that still allows us try and guess two numbers. This modification uses:

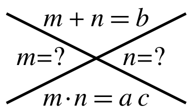
$$a x^2 + b x + c = \frac{1}{a}(a x + m)(a x + n)$$

We are still looking for m and n but now::

1. $m + n = b$

2. $m \cdot n = a c$

Graphically, the modified rule looks like:



Let's do an example with $2x^2 + x - 10$:

$$m + n = 1$$
$$m \cdot n = (2)(-10) = -20$$

Our two numbers are 4 and -5:

$$2x^2 + x - 10 = \frac{1}{2}(2x + m)(2x + n) = \frac{1}{2}(2x + 4)(2x - 5)$$

This can be simplified by factoring out a 2 from $(2x + 4) = 2(x + 2)$:

$$2x^2 + x - 10 = \frac{1}{2}(2x + 4)(2x - 5) = \frac{1}{2} \cdot 2(x + 2)(2x - 5) = (x + 2)(2x - 5)$$