Obtaining Summation Notation

In the previous set of notes, we looked at how we can use summation notation as a short-hand way of representing a long sum of numbers. Then, in the example problems, we took sums in summation notation and converted them into long form.

Often, we will want to take a long list of numbers added together and convert them into summation notation. To do this, we are going to need to use the Habit of a Mathematician: Look for Patterns. Let's illustrate the process with an example:

$$3 \cdot 4^1 + 3 \cdot 4^2 + 3 \cdot 4^3 + 3 \cdot 4^4 + 3 \cdot 4^5$$

To rewrite this in summation notation, we need to first find a pattern for the index of the summation. In our list of sums, we need to search for what it going up by 1 (since that's how our index must behave: it always goes up by 1). In this example, it's pretty obvious that the exponent is starting at 1 and going up by one. Let's use i as the name of the index, so

$$i = 1, 2, 3, 4, 5$$

and the start of our summation notation is:

$$\sum_{i=1}^{5}$$

Since the exponents in the list of sums is going up by 1, the argument of the sum must be $3 \cdot 4^i$. So our summation notation is

$$\sum_{i=1}^{5} 3 \cdot 4^i$$

Another Example

Sometimes it's not obvious which part of the list of sums is going up by and the terms in the list have to converted to some other form. Here's an example:

$$9 + 16 + 25 + 36 + 49$$

There's no obvious consecutive integer pattern in the list of sums, but if we recognize the numbers a perfect squares then we can rewrite the sum as:

$$3^2 + 4^2 + 5^2 + 6^2 + 7^2$$

Now we a list where the base of the exponents is being incremented by 1 (from 3 to 7). The summation notation for this expression would therefore be

$$9 + 16 + 25 + 36 + 49 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = \sum_{i=3}^{7} i^2$$