Summation Notation and Function Notation

One of the powerful characteristics of summation notation is that it does not have to be limited to being a short-hand notation for an explicit sum of numbers, such as:

$$\sum_{j=1}^{10} j = 1+2+3+4+5+6+7+8+9+10$$

Summation notation can also be applied to give a short-hand notation for for the sum of a list of function values. For example, let's assume we have some function $f(x)$. We can then use summation notation as follows:

$$\sum_{i=0}^{4} f(i) \;=\; f(0)+f(1)+f(2)+f(3)+f(4)$$

Example

If $f(x) = x^2 - 4$, evaluate:

$$\sum_{j=2}^{5} f(j)$$

Solution

$$\sum_{j=2}^{5} f(j) = f(2)+f(3)+f(4)+f(5) = \left(2^2-4\right)+\left(3^2-4\right)+\left(4^2-4\right)+\left(5^2-4\right) = 0+5+12+21 = 38$$