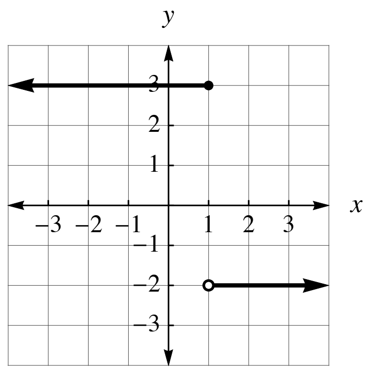
# The Existence of a Limit

In the previous set of notes, we looked at an example where the left-hand limit and the right-hand limit of a function at particular point were not the same.




For this piecewise function, $\lim_{x \to 1^-} f(x) = 3$ and $\lim_{x \to 1^+} f(x) = -2$.

The fact that the left-hand limit is not equal to the right-hand limit raises an interesting question: what is

$$\lim_{x \to 1} f(x) \text{ ?}$$

When the left- and right-hand limits both exist but are not the same, mathematicians have agreed that the limit does not exist (or, DNE). In our example:

$$\lim_{x \to 1} f(x) = \text{DNE}$$

If you are thinking about arguing that the limit should be 3, $\lim_{x \to 1} f(x) = 3$ , since that is the value of the function at $x = 1$, it is really important to keep in mind that a limit does not require the function to exist at the point it is approaching: it does not matter whether $f(1)$ exists! All that matters is whether the function approaches the same value from both directions. In other words,

$$\lim_{x \to a} f(x) \text{ exists if and only if } \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

While this may seem really arbitrary right now, you will soon see why it is so important that a limit is defined in this way.