Introduction to Logarithms

Let's begin with a quick review of exponents:

Base $\longrightarrow 5^7 \longleftarrow$ *Exponent*

Exponents have two parts: a *base* and an *exponent*. The number or variable being multiplied is the *base* and the number of times it is multiplied is the *exponent*. Sometimes the exponent is referred to as the "power" or the "index". For the above example, we would say "five to the seven" or "five raised to the seventh power" or "five raised by the exponent seven". Of course, raising anything to a power is a fundamentally a shortcut for repeated multiplication:

$$5^7 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 78\,125$$
$$a^3 = a \cdot a \cdot a$$

Now let's consider a simple exponent:

$$2^3 = 8$$

This exponent says that 2 (the base) raised to the power of 3 (the exponent) gives us 8. So, if I were to ask the question:

What power of 2 gives 8?

We know the answer is 3. Here is another question:

What power of 3 gives 81?

We could write this question as:

$$3^? = 81$$

or we could use "power" notation:

$$\text{power}_3(81) = ?$$

The last equation can be read as: What power of base 3 yields 81? The answer is 4; that is:

$$3^4 = 81 \quad \text{or} \quad \text{power}_3(81) = 4 \quad \text{or} \quad \text{power}_3\left(3^4\right) = 4$$

Here are a few more examples:

$$\text{power}_2(16) = 4$$
$$\text{power}_5(125) = 3$$
$$\text{power}_{1/2}(4) = -2$$

While it is helpful to keep in mind that our notation is all about powers of some base number, mathematicians don't use the word power; instead, they use the word *logarithm* and abbreviate it in equations as "log":

$$\log_2(16) = 4$$
$$\log_5(125) = 3$$
$$\log_{1/2}(4) = -2$$

By now, you may have noticed a pattern:

$$\log_2(16) = 4 = \log_2\left(2^4\right)$$
$$\log_5(125) = 3 = \log_5\left(5^3\right)$$
$$\log_{1/2}(4) = -2 = \log_{1/2}\left((1/2)^{-2}\right)$$

The pattern is: logs undo exponents!

$\log_b(b^a) = a \qquad \text{Example}: \quad \log_{10}\left(10^5\right) = 5$

If we take a base b and raise it to the power of a, then taking the logarithm of that (to the base b) gives us a. This also works in the other direction:

$b^{\log_b(a)} = a \qquad \text{Example}: \quad 10^{\log_{10}(100)} = 10^{\log_{10}\left(10^2\right)} = 10^2 = 100$

Exponents undo logs!