Properties of Logarithms

Since logarithms "undo" exponents (or to be mathematically correct: logarithmic and exponential functions are inverse functions), there are properties of logarithms that can be discovered by using this relationship between logarithms and exponents.

The Product Property

One property of exponents is how the exponents add:

$$b^p\, b^q = b^{p+q}$$

If we take the logarithm of both sides of that equation we get:

$$\log_b(b^p\, b^q) = \log_b(b^{p+q})$$

The right-hand side of this equation reduces to:

$$\log_b(b^p\, b^q) == p + q$$

Let's let $m = b^p$ and $n = b^q$, so we get:

$$\log_b(m \cdot n) = p + q$$

But, if we take the log of m we get

$$\log_b(m) = \log_b(b^p) = p$$

And if we take the log of n we get

$$\log_b(n) = \log_b(b^q) = q$$

This means that

$$\log_b(m \cdot n) = \log_b(m) + \log_b(n)$$

In words, the product property of logarithms says that the log of a product of terms is the sum of the logs of the terms.

The Quotient Property

Another property of exponents is:

$$\frac{b^p}{b^q} = b^{p-q}$$

If we repeat the process that we used for the Addition Property we can write:

$$\log_b\left(\frac{b^p}{b^q}\right) = \log_b(b^{p-q}) = p - q$$

Using the same definitions of m and n we get

$$\log_b\left(\frac{m}{n}\right) = p - q$$

So

$$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$$

In words, the quotient property of logarithms says that the log of a quotient of terms is the difference of the logs of the terms.

The Power Property

Another property of exponents is:

$$(b^p)^q = b^{p \cdot q}$$

Again, let's take the logs of both sides of this equation:

$$\log_b((b^p)^q) = \log_b(b^{p \cdot q}) = p \cdot q$$

Let's also use the same definitions of m:

$$m = b^p$$

As before:

$$\log_b(m) = p$$

So

$$\log_b(m^q) = p \cdot q = q \cdot p$$

And

$$\log_b(m^q) = q \log_b(m)$$

In words, the power property of logarithms says that the log of an exponential term is the product of the exponent and the base.