Change of Base

If you take a look at calculator, you'll see that it (most likely) only has two log buttons: one for common logs (base 10) and one for natural logs (base e). In some problems involving logs, the base is not 10 or e but you still need to calculate its value. For example,

$$\log_4(17) = ?$$

To calculate this, we want to change the base from 4 to either 10 or e (it doesn't matter which if our calculator handles both). In other words, we want to write our log expression into the form:

$$\log_4(17) = k \cdot \log(17) \quad \text{or} \quad j \cdot \ln(17)$$

where j and k are "correction factors" to give us the right answer. Let's do an example:

$$y = \log_4(17)$$

Let's get rid of the base 4 by using exponents:

$$4^y = 4^{\log_4(17)} = 17$$

Now lets take the common log of both sides:

$$\log(4^y) = \log(17)$$

If we use the Power Property we get

$$y \log(4) = \log(17)$$

If we divide both sides by log(4) we get our answer:

$$y = \log_4(17) = \frac{\log(17)}{\log(4)} = 2.0437$$

In this example, $k = 1 / \log(4)$. Be sure to notice where the number 4 came from: it's the original base.

Generalizing

The Change of Base Rule for Logarithms can be generalized to convert between any two bases as follows. Let's start with a logarithm with base b that we want to covert to a logarithm with base a:

$$y = \log_b x$$

As before, let's first get rid of the base b:

$$b^y = x$$

Next, let's take the logarithm of both sides (using base a):

$$\log_a(b^y) = \log_a x$$

Now apply the Power Property:

$$y \log_a b = \log_a x$$

Finally, we solve for y:

$$y = \frac{\log_a x}{\log_a b}$$

So the Change of Base Rule for logarithms is:

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Example

Convert $\log_6(x)$ to base 2.

Solution

Using the Change of Base Rule

$$\log_6(x) = \frac{\log_2(x)}{\log_2(6)}$$

To calculate $\log_2(6)$ we need to use the Change of Base Rule again and convert to a natural or common log:

$$\log_2(6) = \frac{\log(6)}{\log(2)} = 2.5850$$

So,

$$\log_6(x) = \frac{\log_2(x)}{\log_2(6)} = \frac{\log_2(x)}{2.580} = 0.3868528\,\log_2(x)$$