Condensing Logarithmic Expressions

The reverse of expanding a logarithmic expression is called condensing. In general, the order that we would have used to expand an expression is reversed in order to condense it.

Using the Power Property

Let's start with a simple example using the Power Property:

$$\frac{1}{3}\log(x) = \log\left(x^{1/3}\right)$$

While we could have converted the fractional exponent, 1/3, into a radical sign, that's rarely needed.

Using the Product Property

If we have log terms being added, we can condense using the Product Property

$$\log_2(x) + \log_2(y) = \log_2(x\,y)$$

Using the Quotient Property

If we have log terms being subtracted, we can condense using the Quotient Property

$$\log_7(a) - \log_7(b) = \log_7\left(\frac{a}{b}\right)$$

Using Combinations of Properties

Condensing often requires more than one operation and multiple properties. For example,

$$3\ln(x) + 2\ln(z) = \ln\left(x^3\right) + \ln\left(z^2\right) = \ln\left(x^3\,z^2\right)$$

Here is another example:

$$\frac{1}{3}\log_3(11) - \frac{1}{2}\log_3(5) + \log_3(x) = \log_3\left(11^{1/3}\right) - \log_3\left(5^{1/2}\right) + \log_3(x) = \log_3\left(\frac{11^{1/3}\,x}{5^{1/2}}\right)$$

Here is a more complicated example:

$$\frac{1}{5}(\log_3 x + \log_3 y - \log_3 z) - 2\log_3(x-5) - 4\log_3 z - \log_3(16)$$

$$\frac{1}{5}\left(\log_3\left(\frac{x\,y}{z}\right)\right) - (2\log_3(x-5) + 4\log_3 z + \log_3(16))$$

$$\log_3\left(\frac{x\,y}{z}\right)^{1/5} - \left(\log_3(x-5)^2 + \log_3 z^4 + \log_3(16)\right)$$

$$\log_3\left(\frac{x\,y}{z}\right)^{1/5} - \log_3\left(16\,(x-5)^2\,z^4\right)$$

$$\log_3\left(\frac{x^{1/5}\,y^{1/5}}{16\,z^{1/5}\,z^4(x-5)^2}\right) = \log_3\left(\frac{x^{1/5}\,y^{1/5}}{16\,z^{21/5}(x-5)^2}\right)$$

[Challenge] Change of Base

Be careful when the logs have different bases, such as

$$\log_3(x) + 2\log_2(x)$$

The first step is to change the base (typically to either common or natural log). Let's convert to common logs using the Change of Base Rule:

$$\log_3(x) + 2\log_2(x) = \frac{\log(x)}{\log(3)} + 2\,\frac{\log(x)}{\log(2)}$$

We can now proceed to condense using the Power and Product Properties:

$$\log_3(x) + 2\log_2(x) = \frac{\log(x)}{\log(3)} + 2\,\frac{\log(x)}{\log(2)} = \log\left(x^{1/\log(3)}\right) + \log\left(x^{2/\log(2)}\right) = \log\left(x^{1/\log(3)}\,x^{2/\log(2)}\right) = \log\left(x^{\frac{1}{\log(3)} + \frac{2}{\log(2)}}\right)$$

Note that we could have factored first,

$$\log_3(x) + 2\log_2(x) = \frac{\log(x)}{\log(3)} + 2\,\frac{\log(x)}{\log(2)} = \left(\frac{1}{\log(3)} + \frac{2}{\log(2)}\right)\log(x)$$

and achieved the same result.