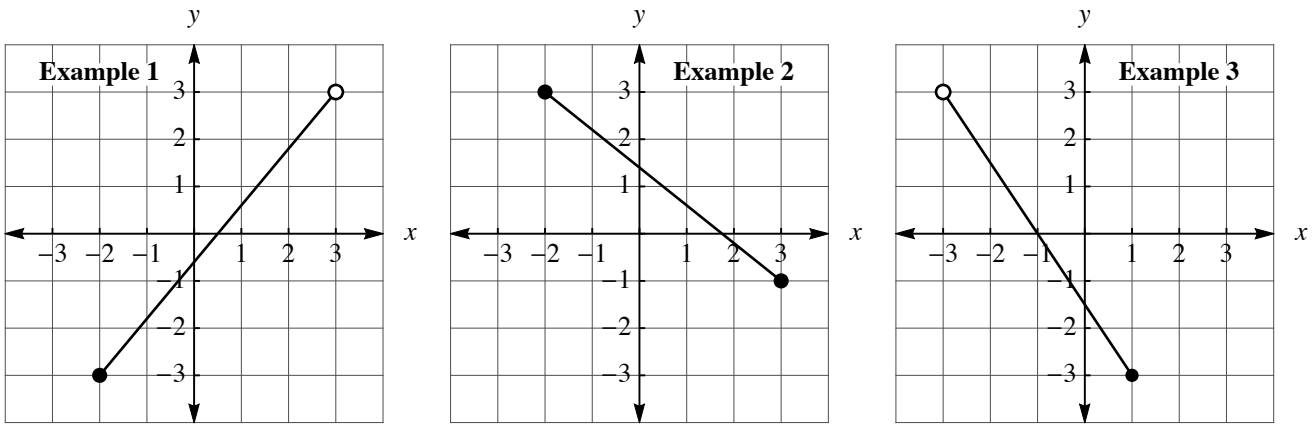
The Equation of a Line Segment

Obtaining the equation of a line segment is very similar to finding the equation of a ray, and it has pretty much the same steps. This will be illustrated using three examples:



Example 1

Step 1

Find the equation of the corresponding line.

From the graph, one point on the line is $(-2, -3)$ so

$$x_1 = -2 \quad \text{and} \quad y_2 = -3$$

Another point is $(3, 3)$. So the slope is

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{6}{5}$$

Using the point-slope form of the equation of the line, we can get its equation:

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = \frac{6}{5}(x - (-2))$$
$$y + 3 = \frac{6}{5}x + \frac{12}{5}$$
$$y = \frac{6}{5}x + \frac{12}{5} - \frac{15}{5}$$
$$y = \frac{6}{5}x - \frac{3}{5}$$

Step 2

Find the inequality that describes the valid x-values. The valid x values are all values greater than and equal to -2 but less than (and not equal to!) 3. We can write this as:

$$x \geq -2 \quad \text{and} \quad x < 3$$

We can combine these two inequalities into a compound inequality:

$$-2 \leq x < 3$$

Step 3

Put the equation and inequality together into a single statement:

$$y = \frac{6}{5}x - \frac{3}{5} \text{ and } -2 \le x < 3$$

Example 2

Step 1

Find the equation of the corresponding line.

From the graph, one point on the line is $(-2, 3)$ so

$$x_1 = -2 \quad \text{and} \quad y_2 = 3$$

Another point is $(3, -1)$. So the slope is

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-4}{5}$$

Using the point-slope form of the equation of the line, we can get its equation:

$$y - y_1 = m(x - x_1)$$
$$y - (3) = -\frac{4}{5}(x - (-2))$$
$$y - 3 = -\frac{4}{5}(x + 2)$$
$$y = -\frac{4}{5}x - \frac{8}{5} + \frac{15}{5}$$
$$y = -\frac{4}{5}x + \frac{7}{5}$$

Step 2

Find the inequality that describes the valid x-values. The valid x values are all values greater than and equal to -2 but less than and equal to 3. We can write this as:

$$x \ge -2 \quad \text{and} \quad x \le 3$$

We can combine these two inequalities into a compound inequality:

$$-2 \le x \le 3$$

Step 3

Put the equation and inequality together into a single statement:

$$y = -\frac{4}{5}x + \frac{7}{5} \text{ and } -2 \le x \le 3$$

Example 3

Step 1

Find the equation of the corresponding line.

From the graph, one point on the line is $(-3, 3)$ so

$$x_1 = -3 \quad \text{and} \quad y_2 = 3$$

Another point is $(1, -3)$. So the slope is

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-6}{4} = -\frac{3}{2}$$

Using the point-slope form of the equation of the line, we can get its equation:

$$y - y_1 = m(x - x_1)$$
$$y - (3) = -\frac{3}{2}(x - (-3))$$
$$y - 3 = -\frac{3}{2}x - \frac{9}{2}$$
$$y = -\frac{3}{2}x - \frac{9}{2} + \frac{6}{2}$$
$$y = -\frac{3}{2}x - \frac{3}{2}$$

Step 2

Find the inequality that describes the valid x-values. The valid x values are all values greater than (but not equal to!) -3 and less than and equal to 1. We can write this as:

$$x > -3 \quad \text{and} \quad x \leq 1$$

We can combine these two inequalities into a compound inequality:

$$-3 < x \leq 1$$

Step 3

Put the equation and inequality together into a single statement:

$$y = -\frac{3}{2}x - \frac{3}{2} \quad \text{and} \quad -3 < x \leq 1$$