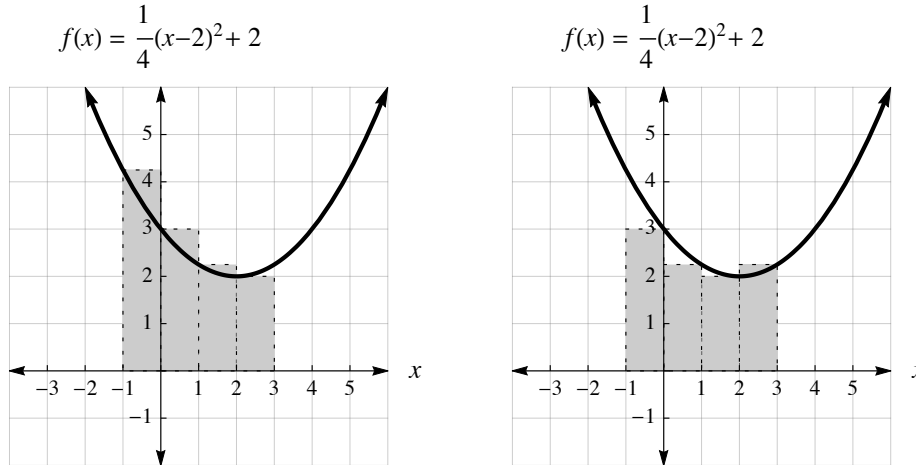


Approximating Area: Left Endpoints versus Right Endpoints

In the previous set of notes, we approximated the area under a curve using rectangles that had their upper left corners on the curve (which allowed us to find the heights of those rectangles so we could then find their areas). It is also possible to draw the rectangles so that the upper right corners are on the curve:



The upper left corners of the rectangles are also known as *left endpoints* and the upper right corners are also known as *right endpoints*.

If we repeat our area calculation using the right endpoints, we get

Rectangle	x	$f(x) = \frac{1}{4}(x-2)^2 + 2$	$A = l \cdot w$
1	0	$f(0) = \frac{1}{4}(0-2)^2 + 2 = 3$	3
2	1	$f(1) = \frac{1}{4}(1-2)^2 + 2 = \frac{9}{4}$	$\frac{9}{4}$
3	2	$f(2) = \frac{1}{4}(2-2)^2 + 2 = 2$	2
4	3	$f(3) = \frac{1}{4}(3-2)^2 + 2 = \frac{9}{4}$	$\frac{9}{4}$

Adding up the area of all 4 rectangles gives us

$$A_{\text{total}} = 3 + \frac{9}{4} + 2 + \frac{9}{4} = \frac{19}{2} = 9.5 \text{ square units.}$$