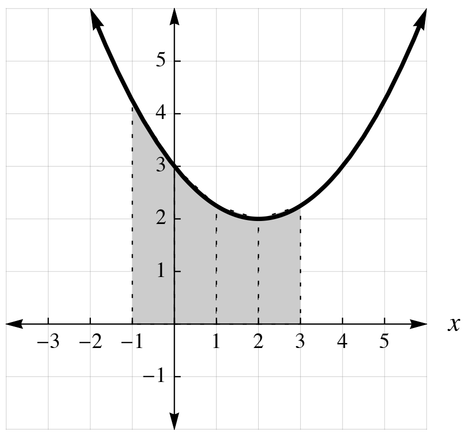
Approximating Area under a Curve with Trapezoids

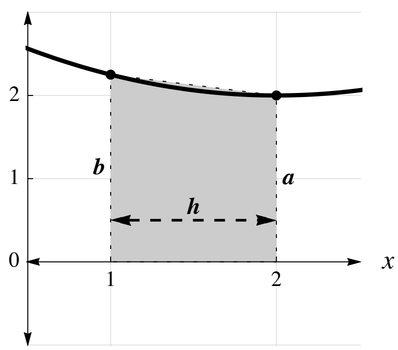
Trapezoids can also be used to approximate the area under a curve. Let's continue with our previous example and find the area of graph of the quadratic equation $f(x) = \frac{1}{4}(x-2)^2 + 2$ from $x = -1$ to $x = 3$ using right trapezoids. Here is the graph:

$$f(x) = \frac{1}{4}(x-2)^2 + 2$$



If we zoom in, we can see better how the trapezoid is approximating the area:

$$f(x) = \frac{1}{4}(x-2)^2 + 2$$



In this example, the height of all the trapezoids is 1. Note how the base of the trapezoid in the above graph has a length $f(1)$:

$$b = f(1)$$

and the length of the other side is $f(2)$:

$$a = f(2)$$

So the area of this trapezoid is:

$$A = \frac{1}{2}(a+b) = \frac{1}{2}(f(1) + f(2))$$

Since

$$f(1) = \frac{1}{4}(1-2)^2 + 2 = \frac{9}{4}$$

and

$$f(2) = \frac{1}{4}(2-2)^2 + 2 = 2$$

the area of this trapezoid is

$$A = \frac{1}{2}(a+b) = \frac{1}{2}(f(1)+f(2)) = \frac{1}{2}\left(\frac{9}{4}+2\right) = \frac{17}{8} \text{ square units}$$

Using a table, we can "stay organized" and find the area of all four trapezoids and get our total area:

Trapezoid	x_{left}	x_{right}	$f(x_{\text{left}})$	$f(x_{\text{right}})$	$A = \frac{1}{2}\left(f(x_{\text{left}}) + f(x_{\text{right}})\right) \cdot h$
1	-1	0	$f(-1) = \frac{17}{4}$	$f(0) = 3$	$\frac{29}{8}$
2	0	1	$f(0) = 3$	$f(1) = \frac{9}{4}$	$\frac{21}{8}$
3	1	2	$f(1) = \frac{9}{4}$	$f(2) = 2$	$\frac{17}{8}$
4	2	3	$f(2) = 2$	$f(3) = \frac{9}{4}$	$\frac{17}{8}$

Adding up the area of all 4 trapezoids gives us

$$A_{\text{total}} = \frac{29}{8} + \frac{21}{8} + \frac{17}{8} + \frac{17}{8} = \frac{21}{2} = 10.5 \text{ square units.}$$

(This is actually very close to the exact answer, which is 31/3 or $10.3\overline{33}$ square units.)