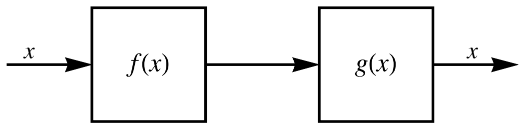
Inverse Functions

Inverse functions are functions that "undo" each other or "cancel" each other out. Using a function machine diagram, inverse functions are represented as:



In other words, if we put some value x into $f(x)$ and then take the output of $f(x)$ and put it into $g(x)$, the output from $g(x)$ will be x: $g(x)$ undoes what $f(x)$ does.

Using composite function notation, we can write the formal mathematical definition of inverse functions:

$$g(f(x)) = f(g(x)) = x$$

Inverse functions are very common and everyone has seen them before. For example:

$f(x) = 2x$ and $g(x) = x/2$

$f(x) = x^2$ and $g(x) = \sqrt{x}$ (for $x \geq 0$)

$\sin(x)$ and $\arcsin(x)$

Example Problem

Show that $f(x) = 4x + 3$ and $g(x) = \frac{x-3}{4}$ are inverse functions.

Solution

To show that f and g are inverse functions we need to show that $g(f(x)) = x$ and $f(g(x)) = x$:

$$g(f(x)) = g(4x+3) = \frac{(4x+3)-3}{4} = \frac{4x+3-3}{4} = \frac{4x}{4} = x$$

and

$$g(f(x)) = f\left(\frac{x-3}{4}\right) = 4\left(\frac{x-3}{4}\right) + 3 = \frac{4}{4}(x-3) + 3 = x - 3 + 3 = x$$

So f and g are inverse functions!

Inverse Function Notation

The inverse of a function, such as $f(x)$, is denoted by $f^{-1}(x)$. This type of notation you have probably seen for the arcsine, arccosine and arctangent functions on your calculator:

$\arcsin(x) = \sin^{-1}(x)$ $\quad$ $\arccos(x) = \cos^{-1}(x)$ $\quad$ $\arctan(x) = \tan^{-1}(x)$

Using this notation, we can write:

$$f\left(f^{-1}(x)\right) = f^{-1}(f(x)) = x$$