Inverse Functions and Reflecting across the line y = x

In the previous set of notes, we saw that we could find the inverse of a function by isolating x (getting x by itself) and then using resulting expression for our inverse function. In other words, we can find an inverse function by "swapping x and y" . Here's an example: find the inverse of $y = x^2 + 2$.

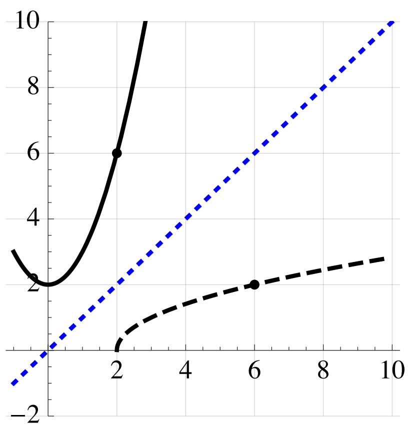
First we solve for x:

$$y - 2 = x^2$$
$$\sqrt{y - 2} = x$$

And then we swap the x and y values

$$y = \sqrt{x - 2}$$

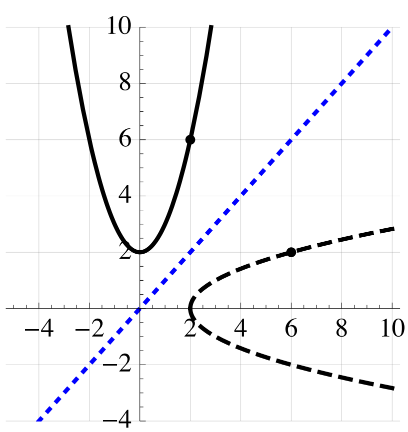
An inverse function, therefore, causes all the points on the original function to get swapped: the x values become the y values and the y values become the x values. If you recall your rigid motion work from Grade 9, this is exactly what happens when we reflect across the line $y = x$: the x values become the y values and the y values become the x values. Therefore, we can visualize an inverse function as being the reflection of the original function across the line $y = x$. Here's the plot of $x^2 + 2$ and $\sqrt{x - 2}$:



Notice how on the original function that when $x = 2$, $y = 6$ and on the inverse function when $x = 6$, $y = 2$.

Domain Restriction (Challenge)

The function $y = x^2 + 2$, graphed above, has a domain of all real numbers (which includes negative x values). If we were to include the negative numbers, our reflected graph would not be a function, as shown in this plot:



In order for the inverse to be a function, we must restrict the domain of the original function to be $D = (0, +\infty)$.