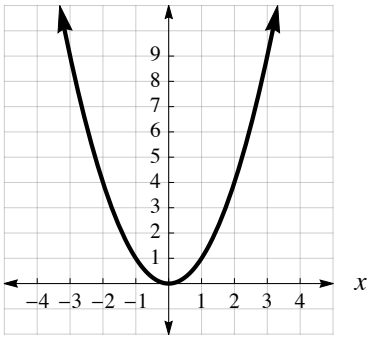
Finite Difference Graphs

A *finite difference* graph is a shows how the slope of a series of secant lines changes on the graph of a function. This let's us see visually how the slope of the function changes as x increases from left to right. It is usually the case the we calculate the slopes with $\Delta x = 1$, so that what we are graphing is simply Δy (but this is not always the case).

Finite Difference Graph for $f(x) = x^2$

To draw a finite difference graph, the first step is to graph the function (in Desmos, for example), to get a sense of what range of x values we want to use. Here's the graph of $f(x) = x^2$:



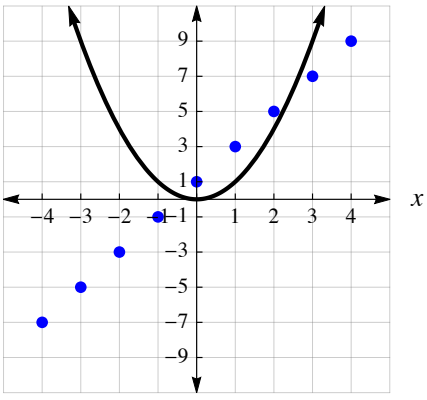
Based on this graph, let's use x from -4 to 4. Here is a table of the x and corresponding y values:

x	−4	−3	−2	−1	0	1	2	3	4	5
$f(x)$	16	9	4	1	0	1	4	9	16	25

The next step is to calculate the finite differences; that is, the differences in the y values. It is very important to be certain you have the correct sign (positive or negative) for the differences. Here is the same table with the differences added:

x	−4	−3	−2	−1	0	1	2	3	4	5
$f(x)$	16	9	4	1	0	1	4	9	16	25
Δy	−7	−5	−3	−1	1	3	5	7	9	

The final step is to graph the finite differences. We have a set of y values (our finite differences) but to plot them, we need corresponding x values. Notice how the $\Delta y = -7$ finite difference is associated with $x = -4$ and $x = -3$. That means we could choose the corresponding x value to be anywhere from -4 to -3. Let's choose $x = -4$ so our first point on the finite difference graph will be $(-4, -7)$, the next point will be $(-3, -5)$, and so on. The graph is shown to the right.



From this graph, we can see how the finite differences are changing as x increases from left to right: the finite differences start at negative values and then increase linearly. We can see this linear increase by looking at how the Δy values are changing:

x	−4	−3	−2	−1	0	1	2	3	4	5
$f(x)$	16	9	4	1	0	1	4	9	16	25
Δy	−7	−5	−3	−1	1	3	5	7	9	
$\Delta(\Delta y)$	2	2	2	2	2	2	2	2		