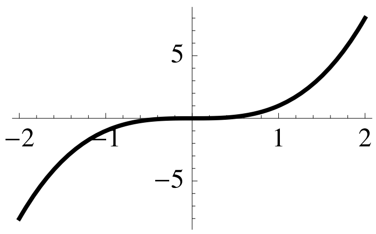
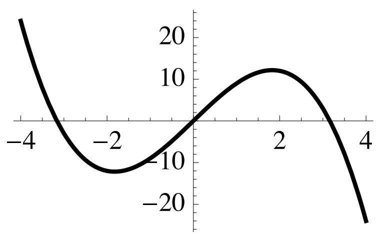
Odd Functions

Functions can be described as *odd*, *even* or neither.

A cubic equation of the form $f(x) = a\,x^3 + b\,x$ is an example of an *even* function. Here is the graph of $f(x) = x^3$:



Notice how the curve above has a rotational symmetry about the origin: all odd functions have a rotational symmetry about the origin. Here is another example, $f(x) = -x^3 + 10\,x$:



When you studied rigid motion in Grade 9 (translation, rotation, and reflection), you would have studied what happens to the x- and y-values to a graph when it is rotated about the origin:

$$(x, y) \Longrightarrow (-x, -y)$$

In words, the reflection causes the x- and y-values to be multiplied by -1(positive x values become negative, positive y-values become negative, and vice versa). *[Recall: Rotation about the origin is the same as reflection across the y-axis and then a reflection across the x-axis]*. Another way of stating this using function notation is that all odd functions have the property:

$$f(-x) = -f(x)$$

A function does not have to a polynomial for it to be an odd function. Rational functions and trig functions can also be even functions: