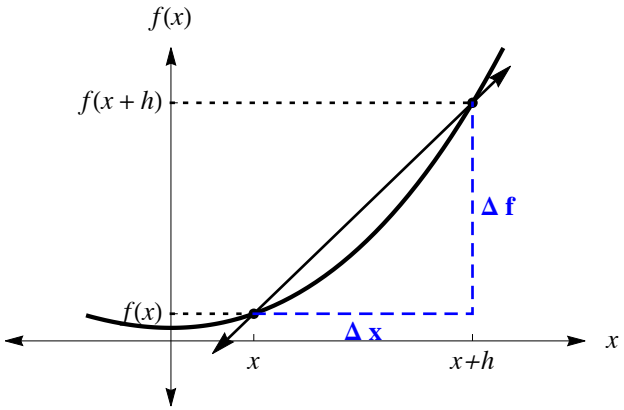
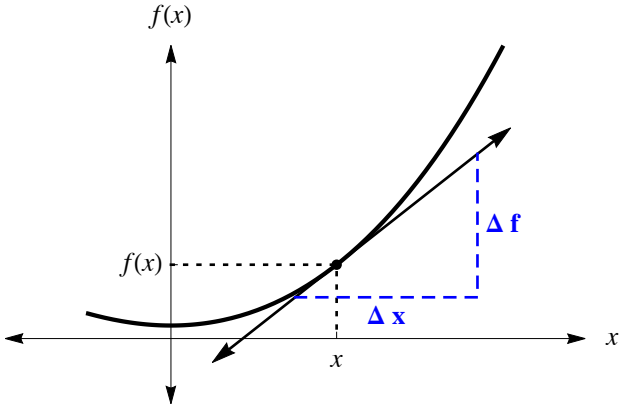
Instantaneous Rate of Change (IROC)

If you can imagine being in a car on the on-ramp to a free-way, you know that when the light changes from red to green, you begin to accelerate from 0 miles per hour (mph) to, say, 60 mph. Let's say that the time to accelerate from 0 to 60 mph took 10 seconds. During those 10 seconds, you experienced every speed from 0 to 60 mph. In fact, if you had used a timer and your speedometer, you could have determined your speed at any point in time during those 10 seconds of acceleration. Your speed at any point in time is called your *instantaneous speed*.

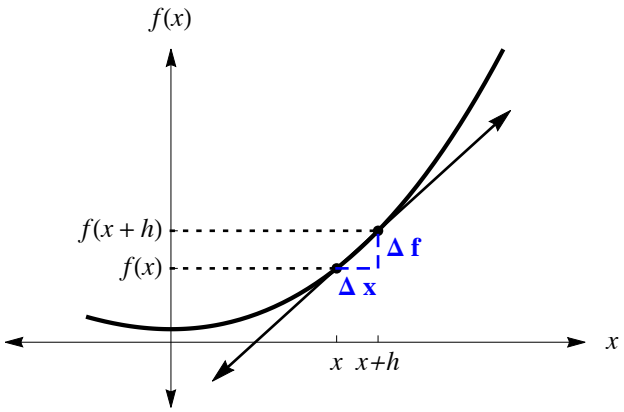
Speed, of course is just one type of rate of change, as we discussed in the previous set of notes. Another example of rate of change is the boiling of water in kettle. The rate of change of the temperature is not necessarily a constant, so at each instant in time, the temperature could be changing at a different rate, which we refer to as the *instantaneous rate of change* (IROC). Recall, the average rate of change is the slope of the secant line between two points on the curve:



Instantaneous rate of change is the slope of the tangent line at a point on the curve:



In the previous set of notes on Limits, we described a method for estimating instantaneous speed (rate of change) using average speed: choose a second point very close to the point of interest and estimate the instantaneous speed using average speed between those two points in time:



From the previous set of notes, we know that the average rate of change (AROC) is:

$$\text{AROC} = m = \frac{\text{rise}}{\text{run}} = \frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

If we let h get really small (let h approach 0), then our AROC estimate for instantaneous rate of change (IROC) becomes more and more accurate. In the limit, as $h \to 0$, AROC equals IROC:

$$\text{IROC} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

In other words, we can find the instantaneous rate of change at some point in time by using limits to find the exact slope of the tangent line at that point in time.