

Homework #6

First & Last Name: _____

Class: _____

For homework to be graded, it must be *fully completed*. This means you must **show your work**.

1. Let $g(x) = \begin{cases} 2 & \text{for } 0 \leq x \leq 3 \\ 3 & \text{for } 3 < x \leq 5 \\ 7 & \text{for } 5 < x \leq 8 \end{cases}$

- a. Sketch the graph of $y = g(x)$. Is this function continuous?
- b. Shade the area between g and the x -axis. What is the shaded area (how many square units)?
- c. g is an example of a **step function**. Why do you think it is called a step function?

2. Given the functions below, compute the following values.

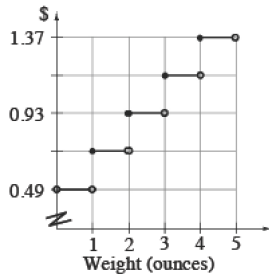
a. $f(x) = \begin{cases} 4 - 3x & \text{for } x \leq 1 \\ x^2 & \text{for } x > 1 \end{cases}$ Calculate $f(0)$, $f(1)$, and $f(3)$.

b. $g(x) = \begin{cases} \sqrt{x} & \text{for } x < 3 \\ 3 - x & \text{for } x \geq 3 \end{cases}$ Calculate $g(1)$, $g(3)$, and $g(9.4)$.

c. $h(x) = \begin{cases} -x & \text{for } x \leq 0 \\ \frac{5}{x} & \text{for } 0 < x \leq 1 \\ 6 - 2x & \text{for } x > 1 \end{cases}$ Calculate $h(-3)$, $h(0)$, $h(0.5)$ and $h(4)$.

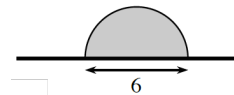
d. Sketch a graph of $y = h(x)$.

3. In order to mail a letter in the United States, postage must be paid based on the weight of the letter. Although rates are tied to the number of ounces, the U.S. Post Office does not allow for payments of partial ounces. A graph showing the postage rates for letters weighing fewer than 5 ounces for the year 2015 is shown below. This is another example of a step function.



- a. How much would you pay for a letter weighing 2.9 ounces? For 3 ounces? For 3.1 ounces?
- b. Write a piecewise-defined function that determines the postage rates for letters weighing between 0 and 5 ounces. Let x represent the weight in ounces, and y represent cost in dollars.

4. A semi-circular flag is shown attached to a “pole” :



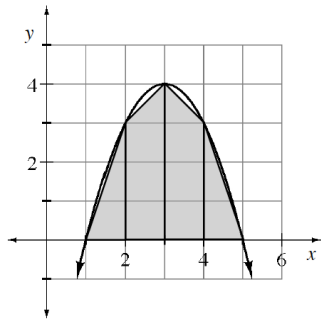
- a. Imagine rotating the flag about its pole and describe the resulting three-dimensional figure. Draw a picture of this figure on your paper. To help you visualize this, use [Desmos](https://www.desmos.com/calculator/dufbj3d7p6) (desmos.com/calculator/dufbj3d7p6).
- b. Calculate the volume of the rotated flag.

5. Sketch a graph of the piecewise-defined function $g(x) = \begin{cases} 2^x & \text{for } x \leq 2 \\ 3x - 2 & \text{for } x > 2 \end{cases}$.

[Desmos](https://www.desmos.com/calculator/cqvtvpg7q9) (desmos.com/calculator/cqvtvpg7q9).

- a. State the domain and range of g .
- b. Is g continuous at $x = 2$? Explain.
- c. Is g continuous for all values of x ?

6. The parabola $y = -(x - 3)^2 + 4$ is graphed below. Four trapezoids of equal width are inscribed for $1 \leq x \leq 5$.



- a. Use the combined area of these trapezoids to approximate the area under the parabola for $1 \leq x \leq 5$.
 - b. Is this area greater or less than the true area under the parabola? Explain.
7. [Challenge] What is the exact value of each of the following trig expressions?
- a. $\sin\left(\frac{5\pi}{3}\right)$
 - b. $\tan\left(\frac{7\pi}{6}\right)$
 - c. $\sec\left(\frac{5\pi}{4}\right)$
 - d. $\csc(\pi)$
8. [Challenge] Sketch the graph of $y = \frac{1}{x}$. [Desmos](https://www.desmos.com/calculator/ihninztuyq) (desmos.com/calculator/ihninztuyq).
- a. Why does this graph have a vertical asymptote? What is the equation of that asymptote?
 - b. State the equation of the horizontal asymptote.
 - c. Alter the equation $y = \frac{1}{x}$ so that the vertical asymptote is $x = 1$ and the horizontal asymptote is $y = 3$.
9. [Challenge] Use polynomial division to rewrite each of the following rational expressions.
- a. $\frac{x^3+2x^2-3x+4}{x+3}$
 - b. $\frac{x^4-5x^2+3x-3}{x-2}$