Homework Solutions #1

1. Determine the intercepts of the line $4x - 5y = 11$.

The x-intercept occurs when $y = 0$:

$$4x - 5(0) = 11$$
$$4x = 11$$
$$x = \frac{11}{4} \quad \Longrightarrow \quad \left(\frac{11}{4}, 0\right)$$

The y-intercept occurs when $x = 0$:

$$4(0) - 5y = 11$$
$$-5y = 11$$
$$y = \frac{-11}{5} \quad \Longrightarrow \quad \left(0, -\frac{11}{5}\right)$$

2. What is the equation of the line that has an x-intercept of -3 and a y-intercept of -8? Write your answer in both slope-intercept and point-slope forms.

The intercepts are $(-3, 0)$ and $(0, -8)$, which are points on the line. The slope of the line is:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8 - 0}{0 - (-3)} = -\frac{8}{3}$$

Point-slope form is: $y - y_1 = m(x - x_1)$ or $y - y_2 = m(x - x_2)$:

$$y - 0 = -\frac{8}{3}(x - (-3)) \quad \Longrightarrow \quad y = -\frac{8}{3}(x + 3) \quad \text{or}$$

$$y - (-8) = -\frac{8}{3}(x - 0) \quad \Longrightarrow \quad y + 8 = -\frac{8}{3}x$$

Slope-intercept form is $y = mx + b$, where b is the y-intercept:

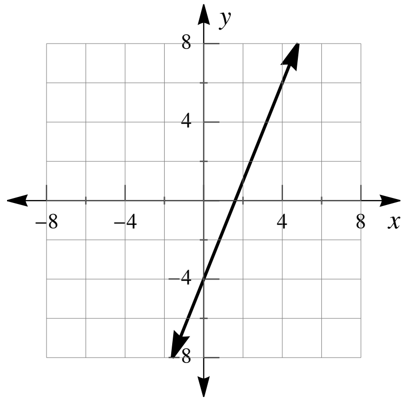
$$y = -\frac{8}{3}x + (-8) \quad \Longrightarrow \quad y = -\frac{8}{3}x - 8$$

3. If a line goes through the points $(3.7, 5.3)$ and $(-7.1, 4.9)$ what is its slope?

The slope is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4.9 - 5.3}{-7.1 - 3.7} = \frac{-0.4}{-10.8} = \frac{4}{108} = \frac{1}{27} \approx 0.0370$$

4. Find the equation of the line in slope-intercept form. Use exact numbers.



Using the graph, two points on the line can be found: $(0, -4)$ and $(4, 6)$. So the y-intercept is $b = -4$ and the slope is:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-4)}{4-0} = \frac{10}{4} = \frac{5}{2}$$

Slope-intercept form is $y = mx + b$, where b is the y-intercept:

$$y = \frac{5}{2}x + (-4) \implies y = \frac{5}{2}x - 4$$

5. If $f(s) = 8\,s - 4$, what is

a. $f(-2) = 8\,(-2) - 4 = -16 - 4 = -20$

b. $f(-1) = 8\,(-1) - 4 = -8 - 4 = -12$

c. $f(0) = 8\,(0) - 4 = 0 - 4 = -4$

d. $f(1) = 8\,(1) - 4 = 8 - 4 = 4$

e. $f(2) = 8\,(2) - 4 = 16 - 4 = 12$

6. [Challenge] If $f(x) = x^2 - 2$ and $g(x) = \sqrt{x} - 2$ what is

a. $g(0) = \sqrt{0} - 2 = -2$, $f(-2) = (-2)^2 - 2 = 4 - 2 = 2$

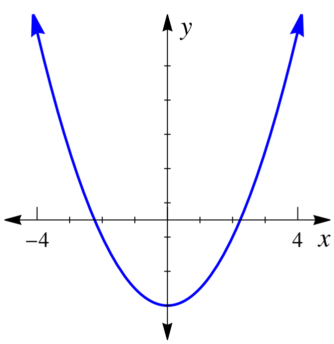
b. $f(0) = 0^2 - 2 = -2$, $g(-2) = \sqrt{-2} - 2 =$ Undefined (square root of a negative number is not a real number)

c. $f(r) = r^2 - 2$, $g\left(r^2 - 2\right) = \sqrt{r^2 - 2} - 2 = \left(r^2 - 2\right)^{1/2} - 2$

d. $g(s) = \sqrt{s} - 2$, $f\left(\sqrt{s} - 2\right) = \left(s^{1/2} - 2\right)^2 - 2 = \left(s - 2\,s^{1/2} - 2\,s^{1/2} + 4\right) - 2 = s - 4\,s^{1/2} + 2$

7. What is the domain and range of the function $f(x) = x^2 - 5$? Write your answers in both interval notation and set notation.

The function $f(x) = x^2 - 5$ is a quadratic function so its graph will be a parabola:



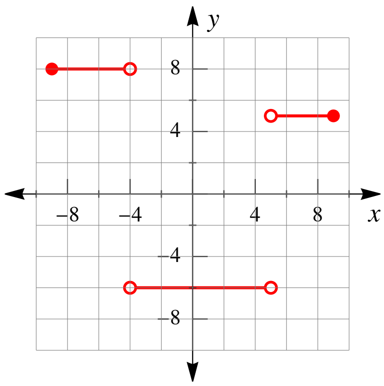
The domain of a quadratic function is all the real numbers:

Interval Notation : $D = (-\infty, \infty)$
Set Notation : $D = \{x : -\infty < x < \infty\}$

Since the parabola opens up, the vertex of the parabola, $(0, -5)$, is the lower bound for the range of the function:

Interval Notation : $R = [-5, \infty)$
Set Notation : $R = \{y : -5 \le y < \infty\}$

8. **[Challenge]** What is the domain and range of the following function? Write your answers in both interval notation and set notation.



Using the graph, the domain and range can be read directly.

The domain of the function is:

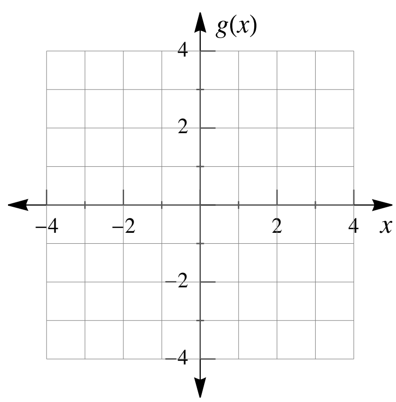
Interval Notation : $D = [-9, -4) \cup (-4, 5) \cup (5, 9]$
Set Notation : $D = \{x : -9 \le x < -4 \, , -4 < x < 5, \; 5 < x \le 9\}$,

The range of the function is:

Interval Notation : $R = [-6, -6] \cup [5, 5] \cup [8, 8]$
Set Notation : $R = \{y : y = -6, 5, 8\}$

9. Graph the piecewise function

$$g(x) = \begin{cases} -x - 3 & x < -2 \\ -2 & -2 \le x < 1 \\ 2x - 4 & x \ge 1 \end{cases}$$



The piecewise function has 3 parts. For the first part, the function is linear and two points on the line are:

At $x = -2$, $g(-2) = -(-2) - 3 = 2 - 3 = -1 \implies (-2, -1)$

At $x = -3$, $g(-3) = -(-3) - 3 = 3 - 3 = 0 \implies (-3, 0)$

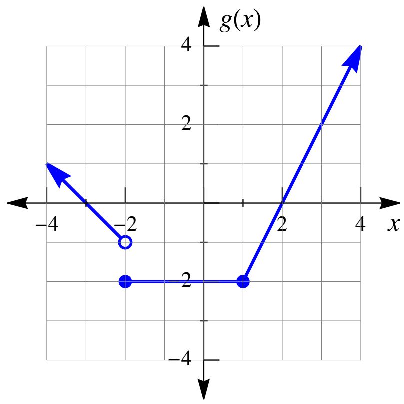
For the second part, the function is a horizontal line, $y = -2$

For the third part, the function is linear and two points on the line are:

At $x = 1$, $g(1) = 2(1) - 4 = 2 - 4 = -2 \implies (1, -2)$

At $x = 2$, $g(2) = 2(2) - 4 = 4 - 4 = 0 \implies (2, 0)$

Using these points, we can sketch the piecewise function:



g(x)
4
2
−4
−2
2
4
x
−2
−4