Homework Solutions #2

1. Rewrite the following expressions in the form b^n.

a. $\frac{y^{10}}{y^{22}} = y^{10-22} = y^{-12}$

b. $\frac{7^{-6}}{7^{11}} = 7^{-6-11} = 7^{-17}$

c. $\frac{t^3}{t^7} = t^{3-7} = t^{-4}$

d. $\left(5^{-1}\right)\left(5^5\right) = 5^{-1+5} = 5^4$

e. $z^{-2} \cdot z^{-17} = z^{-2+(-17)} = z^{-2-17} = z^{-19}$

f. **[Challenge]** $\left(2^{-4}\right)\left(4^{-2}\right) = 2^{-4}\,(2\cdot 2)^{-2} = 2^{-4}\cdot 2^{-2}\cdot 2^{-2} = 2^{-4-2-2} = 2^{-8}$

2. Rewrite the following radical expressions in exponential form.

a. $\sqrt{t^{12}} = \left(t^{12}\right)^{1/2} = t^{12\cdot 1/2} = t^6$

b. $\sqrt[5]{x^3} = \left(x^3\right)^{1/5} = x^{3\cdot 1/5} = x^{3/5}$

c. $\sqrt[4]{1/p^3} = \left(\frac{1}{p^3}\right)^{1/4} = \left(p^{-3}\right)^{1/4} = p^{-3\cdot 1/4} = p^{-3/4}$

d. $\sqrt[6]{(y/x)^{12}} = \left(\left(\frac{y}{x}\right)^{12}\right)^{1/6} = \left(\frac{y}{x}\right)^{12\cdot 1/6} = \left(\frac{y}{x}\right)^2 = \frac{y^2}{x^2} = x^{-2}\,y^2$

3. $M = -6\,f^6 + 3\,f^3\,g^3 - 4\,g^6$ and $N = 4\,f^6 - 5\,f^3\,g^3 - 3\,g^6$.

a. What is $M + M$? $M + M = 2\,M = 2\left(-6\,f^6 + 3\,f^3\,g^3 - 4\,g^6\right) = -12\,f^6 + 6\,f^3\,g^3 - 8\,g^6$

b. What is $M + N$? $M + N = \left(-6\,f^6 + 3\,f^3\,g^3 - 4\,g^6\right) + \left(4\,f^6 - 5\,f^3\,g^3 - 3\,g^6\right) = -2\,f^6 - 2\,f^3\,g^3 - 7\,g^6$

c. What is $M - N$? $M - N = \left(-6\,f^6 + 3\,f^3\,g^3 - 4\,g^6\right) - \left(4\,f^6 - 5\,f^3\,g^3 - 3\,g^6\right) = -10\,f^6 + 8\,f^3\,g^3 - g^6$

d. What is $N - M$? $N - M = -(M - N) = -\left(-10\,f^6 + 8\,f^3\,g^3 - g^6\right) = 10\,f^6 - 8\,f^3\,g^3 + g^6$

e. **[Challenge]** What is $3\,N - 3\,M$? $3\,N - 3\,M = 3\,(N - M) = 3\left(10\,f^6 - 8\,f^3\,g^3 + g^6\right) = 30\,f^6 - 24\,f^3\,g^3 + 3\,g^6$

4. Find the values for c and d that would make the following equation true: $\left(c\,x^6\right)\left(11\,x^d\right) = 121\,x^4$

$\left(c\,x^6\right)\left(11\,x^d\right) = 11\,c\,x^6\,x^d = 11\,c\,x^{6+d} = 121\,x^4$

$11\,c = 121 \implies c = 11$

$6 + d = 4 \implies d = -2$

5. Express the area of the entire rectangle (your answer should be a polynomial in standard form).

$A = l\cdot w = 6\,v\left(4\,v^3 + 2\,v^2 + 8\right) = 6\,v\cdot 4\,v^3 + 6\,v\cdot 2\,v^2 + 6\,v\cdot 8 = 24\,v^4 + 12\,v^3 + 48\,v$

6. Expand (your answer should be a polynomial in standard form).

a. $3\left(p^3 - p^2\,q + q^3\right) = 3\,p^3 - 3\,p^2\,q + 3\,q^3$

b. $-2\,g^4\left(3\,h + 3\,g^3\,h - 5\,g\right) = -2\,g^4\cdot 3\,h + \left(-2\,g^4\right)\cdot 3\,g^3\,h - 5\,g\left(-2\,g^4\right) = -6\,g^4\,h + -6\,g^7\,h + 10\,g^5$

7. Expand and simplify (your answer should be a polynomial in standard form).

a. $(3\,h+1)\left(5\,h^3-6\,h^2+2\right)=$

$$3\,h\left(5\,h^3-6\,h^2+2\right)+1\left(5\,h^3-6\,h^2+2\right)=$$
$$15\,h^4-18\,h^3+6\,h+5\,h^3-6\,h^2+2=$$
$$15\,h^4-13\,h^3-6\,h^2+6\,h+2$$

b. $\left(7\,b^3-3\right)\left(-5\,b^4+1\right)=$

$$7\,b^3\left(-5\,b^4+1\right)-3\left(-5\,b^4+1\right)=$$
$$-35\,b^7+7\,b^3+15\,b^4-3=$$
$$-35\,b^7+15\,b^4+7\,b^3-3$$

8. **[Challenge]** What is the greatest common factor of $13\,a\,b$ and $9\,a^5$?

$13\,a\,b$ is fully factored so the greatest common factor must be their only shared factor: a.

9. **[Challenge]** Factor $x^2+8\,x+12$ as the product of two binomials.

$$n\cdot m=12 \text{ and } n+m=8 \implies n=2 \text{ and } m=6 \implies x^2+8\,x+12=(x+2)\,(x+6)$$

10. **[Challenge]** Factor $q^{16}+8\,q^8+15$ completely.

$$q^{16}+8\,q^8+15=\left(q^8\right)^2+8\cdot q^8+15$$

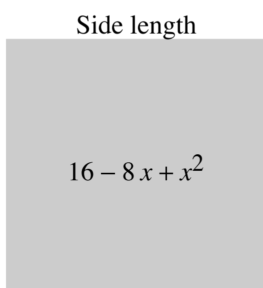
Let $p=q^8$ and substitute:

$$p^2+8\,p+15$$
$$n\cdot m=15 \text{ and } n+m=8 \implies n=3 \text{ and } m=5 \implies p^2+8\,p+12=(p+3)\,(x+5)$$

Substitute $q^8=p$:

$$q^{16}+8\,q^8+15=\left(q^8+3\right)\left(q^8+5\right)$$

11. **[Challenge]** The square below has an area of $16-8\,x+x^2$. What expression represents the length of one side of the square?



If a square has side length s, its area must be $A=s^2=s\cdot s$. So if we factor $16-8\,x+x^2$ into two terms, each term must be a side length. First, let's rewrite $16-8\,x+x^2=x^2-8\,x+16$ and factor:

$$n\cdot m=16 \text{ and } n+m=-8 \implies n=-4 \text{ and } m=-4 \implies x^2-8\,x+16=(x-4)\,(x-4)$$

Side lengths of squares must be positive numbers so it's required that $x-4>0$ or $x>4$. Because the square of a negative number is positive, the factor could also be

$$x^2-8\,x+16=(4-x)\,(4-x) \quad \text{(convince yourself!)}$$

It's still required that the factors (side lengths) are positive numbers, so we require $4-x>0$ or $x<4$. In summary, there are two expressions that represent the length of one side of the square:

$x-4,\ x>4$ or
$4-x,\ x<4$