Continuity 3: Working with Continuous Functions

1. Here is the continuous function we used in the previous handout:

$$f(x) = \begin{cases} 9 - 2^x & x \le 3 \\ \sqrt{x-2} & x > 3 \end{cases}.$$

a. In your own words, explain why f is continuous at $x = 3$

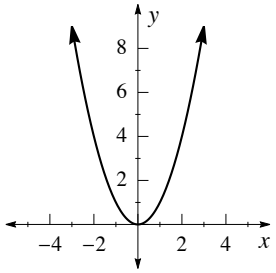
b. Explain why f can also be defined as

$$f(x) = \begin{cases} 9 - 2^x & x < 3 \\ \sqrt{x-2} & x \ge 3 \end{cases}.$$

2. With respect to continuity, functions (such as $f(x)$) can be divided into three categories:

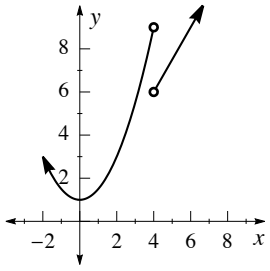
Category I: Continuous for all real values of x

$f(x) = x^2$



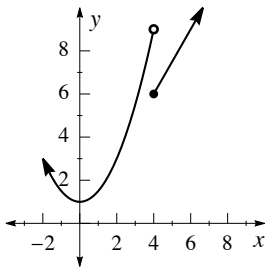
Category II: Continuous on its domain only

$$g(x) = \begin{cases} 0.5x^2 + 1 & x < 4 \\ 1.5x & x > 4 \end{cases}$$



Category III: Discontinuous on its domain.

$$g(x) = \begin{cases} 0.5x^2 + 1 & x < 4 \\ 1.5x & x \ge 4 \end{cases}$$



Using Desmos to help, sketch a graph each of the following functions and decide which category they belong to:

a. $y = \sin(x)$

b. $y = \ln(x)$

c. $y = \sqrt{x}$

d. $y = \lfloor x \rfloor = [\![x]\!]$ (the greatest integer function, which is also called the floor function: the largest integer that is less than or equal to x)

e. $y = x^{1/3}$

f. $y = \arctan(x) = \tan^{-1}(x)$

g. $y = 1/x$

h. $y = \sec(x)$

i. $y = x^3$

j. $y = e^x$

k. $y = \tan(x)$

l. **[Challenge]** $y = \dfrac{x^2}{x}$