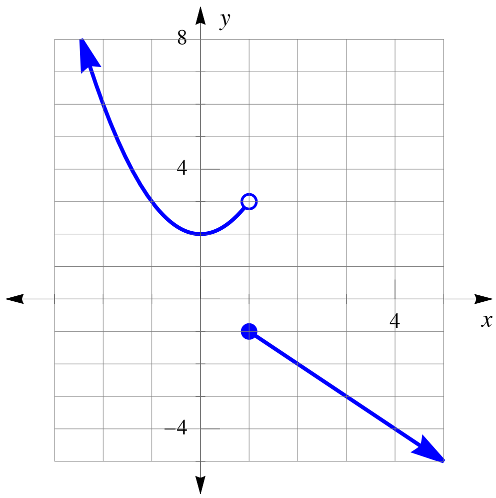
Continuity 5: More Working With Continuity

1. Here is a piecewise function:

$$f(x) = \begin{cases} x^2 + 2 & x < 1 \\ -x & x \geq 1 \end{cases}.$$

a. Sketch a graph of $y = f(x)$. Use Desmos if you need help.



b. Describe, in your own words, how you could modify a piece of the function to make it continuous or **[Challenge]** modify one piece of the function to make it continuous and write your new piecewise equation.

We could take the top piece and translate it down 4 units or take the bottom piece and translate it up 4 units.

$$f(x) = \begin{cases} x^2 + 2 - 4 & x < 1 \\ -x & x \geq 1 \end{cases} = \begin{cases} x^2 - 2 & x < 1 \\ -x & x \geq 1 \end{cases} \text{ or}$$

$$f(x) = \begin{cases} x^2 + 2 & x < 1 \\ -x + 4 & x \geq 1 \end{cases}$$

2. **[Challenge]** Determine the values of a and b such that g is a continuous function. Use the Desmos Tool to verify your ideas.

$$g(x) = \begin{cases} \sqrt{x+3} & x < 1 \\ a(x-1)^2 + b & 1 \leq x < 3 \\ -x + 2 & x \geq 3 \end{cases}$$

From the first piece we know that when $x = 1$, the function must have a value of

$$g(1) = \sqrt{1+3} = \sqrt{4} = 2$$

From the third piece we know that when $x = 3$, the function must have a value of

$$g(3) = -(3) + 2 = -1$$

This means that for the second piece, the following must be true:

$$g(1) = 2$$
$$a(1-1)^2 + b = 2$$
$$a \cdot 0 + b = 2$$
$$b = 2$$

If $b = 2$, then the following must also be true:

$$g(3) = -1$$
$$a(3-1)^2 + 2 = -1$$
$$4a = -3$$
$$a = \frac{-3}{4}$$