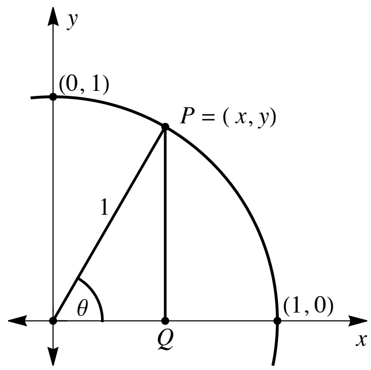
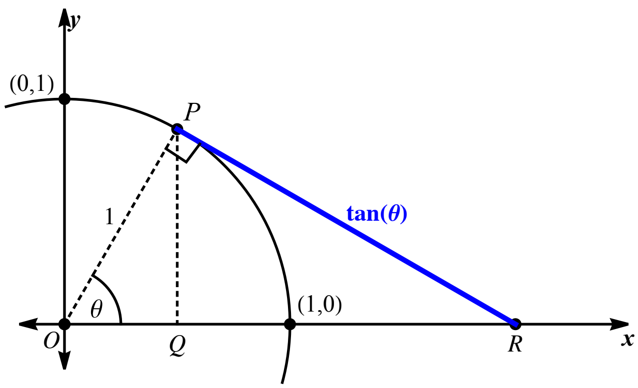
Trigonometry 11: The Definition of the Tangent Function

Consider a Unit Circle with some point (in the first quadrant) with coordinate point $P = (x, y)$. As before, a radial line from this point to the origin (point O) makes an angle θ with the positive x-axis, as shown in the figure below:



Now consider a tangent line that is drawn at point P to the positive x-axis, intersecting the x-axis at point R:



By the Angle-Angle-Angle Theorem (AAA) triangles PQO and RPO are similar triangles. Since corresponding sides of similar triangles are in proportion, we know:

$$\frac{\text{RP}}{\text{PO}} = \frac{\text{PQ}}{\text{QO}}$$

Substituting in for the known side lengths, we get:

$$\frac{\text{RP}}{1} = \frac{y}{x} \implies \text{RP} = \frac{y}{x}$$

This ratio, which is the length RP (highlighted in blue) is known as the tangent of the angle θ and is written

$$\tan\theta = \frac{y}{x}$$

We have already defined that

$$\sin\theta = y \text{ and } \cos\theta = x$$

so,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Because the definition of the **tangent function** is the sine function divided by the cosine, this definition of tan is known as a **quotient identity**. We'll discover a second quotient identity later.