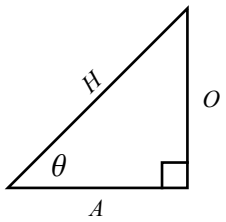
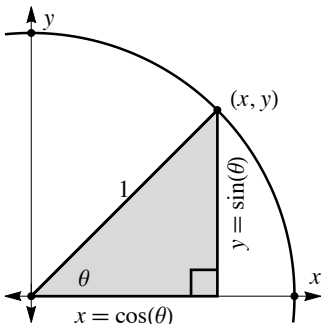
Trigonometry 13: SOH, CAH ,TOA

As we have seen, one of the powerful applications of trigonometry is its ability to give us the side lengths of right angles triangles, when the hypotenuse is on side length 1. But this, of course, is really limited because in real world applications, it is rare that we encounter situations where we are dealing with a right angle triangle and the hypotenuse happens to be 1 unit. What we what to do is use the "Generalize" Habit of a Mathematician to be able to apply our trig functions to right angle triangles with hypotenuses of any length.

To begin, let's draw a triangle we might be interested in, using *H* for hypotenuse, *O* for Opposite and *A* for Adjacent:



Now, lets look at a *similar* triangle (and I really do mean *similar* in the mathematical sense) with a hypotenuse of length 1 unit but with the same angle θ. Notice how I'm labeling the side lengths using the definitions of sine and cosine.



Since these two triangles are similar, their sides must be in proportion. That means:

$$\frac{\sin\theta}{1} = \frac{O}{H} \implies \sin\theta = \frac{O}{H} \qquad \text{SOH}$$

and

$$\frac{\cos\theta}{1} = \frac{A}{H} \implies \cos\theta = \frac{A}{H} \qquad \text{CAH}$$

and

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{O}{A} \implies \tan\theta = \frac{O}{A} \qquad \text{TOA}$$

This is really important! We now have a way of using our sine, cosine and tangent functions on right triangles of any size and shape! For example, find the missing side lengths in the right triangle below.