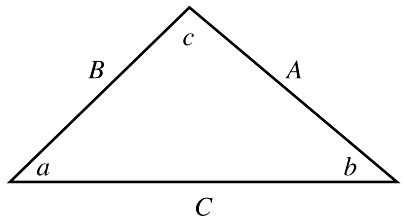
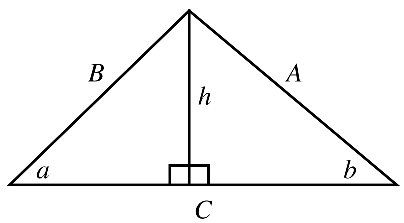
Trigonometry 14: The Law of Sines

Let's take any triangle and label it as shown below (it's intentional that the side with length A is opposite the angle with measure a, side with length B is opposite angle with measure b, and the same for c and C).



Now let's break this triangle into two right triangles by dropping a perpendicular (with length h) from the angle with measure c to the opposite side (with length C):



Using the sine function we know,

$$\sin a = \frac{h}{B} \implies h = B \sin a \qquad \text{and} \qquad \sin b = \frac{h}{A} \implies h = A \sin b$$

This means $B \sin a = A \sin b$, or:

$$\frac{A}{\sin a} = \frac{B}{\sin b}$$

We could repeat this process with the other to angles s to show

$$\frac{A}{\sin a} = \frac{C}{\sin c} \qquad \text{and} \qquad \frac{B}{\sin B} = \frac{C}{\sin c}$$

In summary, the **law of sines** states:

$$\frac{A}{\sin a} = \frac{B}{\sin B} = \frac{C}{\sin C}$$

We can also write this law as the reciprocals:

$$\frac{\sin a}{A} = \frac{\sin B}{B} = \frac{\sin C}{C}$$

This law can be very helpful for finding side lengths for acute and oblique (non-right) triangles. For example, find the missing side lengths in the following triangle: