

## Trigonometry 16: Radians

**Radians** (often abbreviated as *rad*) are just another unit for the measure of an angle. The circumference of a circle with radius  $r$  has length

$$C = 2\pi r$$

This means that there are  $2\pi$  "radius lengths" in the circumference of any circle. If we were to walk the circumference of a circle, after walking  $2\pi$  "radius lengths", we will end up where we started ( $360^\circ$  around the circle). This means:

$$2\pi \text{ "radius lengths" } = 360^\circ$$

Rather than say "radius lengths", the word *radians* is used:  $2\pi \text{ radians} = 360^\circ$

### Converting between Radians and Degrees (and Vice Versa)

$$2\pi \text{ rad} = 360^\circ \implies 1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ$$

To convert from radians to degrees, we take the number of radians and multiply by  $180/\pi$ . Or, if we forget that rule we can always just set up the proportion. For example, let's find the number of radians in  $15^\circ$ :

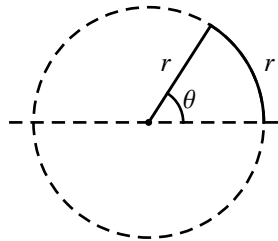
$$\frac{15^\circ}{360^\circ} = \frac{x}{2\pi \text{ rad}} \implies x = \frac{15}{360} \cdot 2\pi = \frac{30\pi}{360} = \frac{\pi}{12} \text{ rad}$$

To convert from radians to degrees, we can use the same proportion. For example, let's find the number of degrees in 5 rad:

$$\frac{x}{360^\circ} = \frac{5 \text{ rad}}{2\pi \text{ rad}} \implies x = 360 \cdot \frac{5}{2\pi} = \frac{900}{\pi} = 286.4789^\circ$$

### An Alternative View of a Radian

A radian is the angle subtended by an arc of a circle that has the same length as the circle's radius, as shown below:



Since the circumference of the circle is  $C = 2\pi r$ , the angle  $\theta$  must be:

$$\frac{r}{C} = \frac{\theta}{360^\circ} \implies \frac{r}{2\pi r} = \frac{\theta}{360^\circ} \quad \theta = 360^\circ \cdot \frac{1}{2\pi} = \frac{180^\circ}{\pi} = 57.29577951^\circ$$