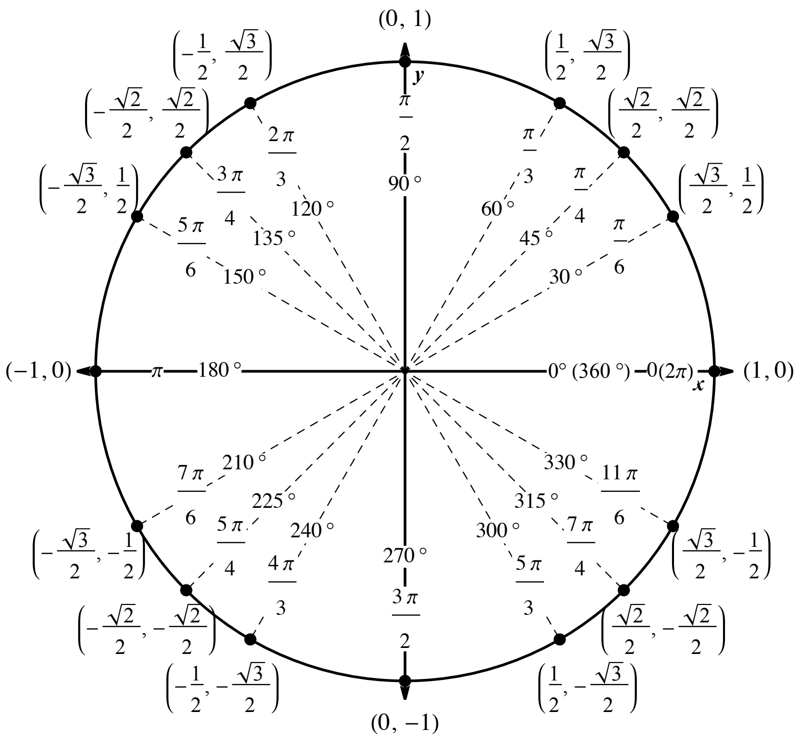
Trigonometry 24: Inverse Functions: Arccosine

Let's revisit the unit circle and ask the question, "At what angle is the cosine equal to 1/2?".



Using the unit circle, we can see that there are two answers to that question: $\theta = 60^o$ and $\theta = 300^o = -60^o$. Another way of stating this is:

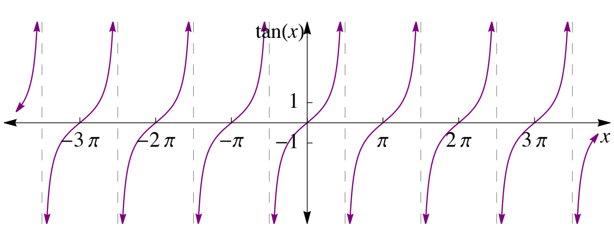
$$\cos 60^o = 1/2 \qquad \text{and} \qquad \cos 300^o = 1/2$$

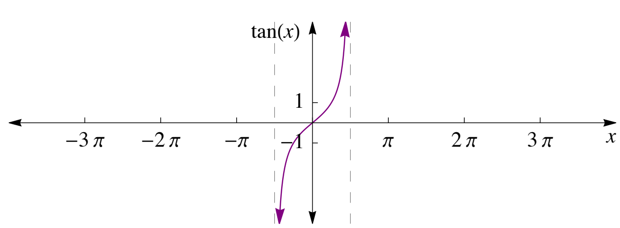
If we were to put our original question in equation form, we could write: "Solve for θ: $\cos\theta = 1/2$". Solving for θ is the inverse of finding the cosine of θ. In trigonometry, the inverse of the cosine function is called the **arccosine function**:

$$\theta = \arccos(x) = \arccos(1/2) = 60^o$$

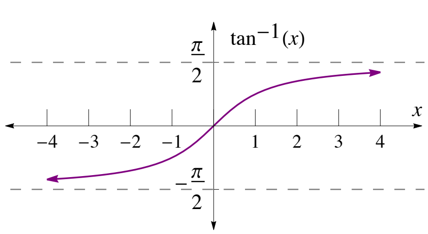
Arccosine is a function, which means that for each input, x, there can only be one output, θ. As we saw above, using the unit circle, there were two outputs: 60^o and 300^o. To get around this problem, the range of the arccosine function is restricted to the Quadrants I and II.

More insight into the arccosine function by looking at the graph of the cosine function:





The tangent function does not pass the horizontal line test; however if we restrict the domain of $\tan(x)$ to $[-\pi/2, \pi/2]$ it does. The function $\arctan(x)$ is the inverse of the tangent function with its domain restricted. Here's it's graph:



Note that $\arctan(x)$ is often written as $\tan^{-1}(x)$.