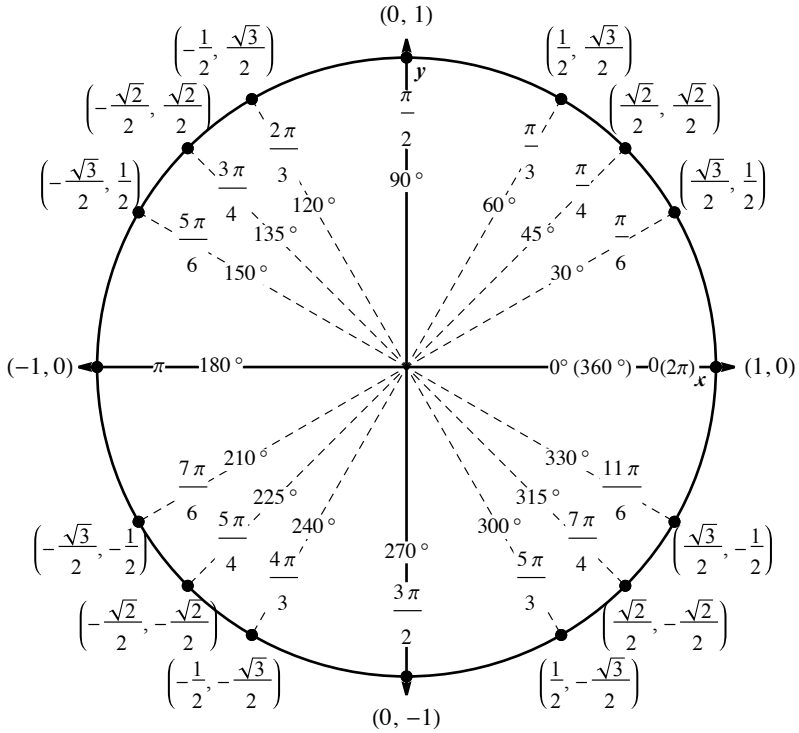
Trigonometry 25: Inverse Functions: Arcsine

Let's revisit the unit circle and ask the question, "At what angle is the sine equal to 1/2?".



Using the unit circle, we can see that there are two answers to that question: $\theta = 30°$ and $\theta = 150°$. Another way of stating this is:

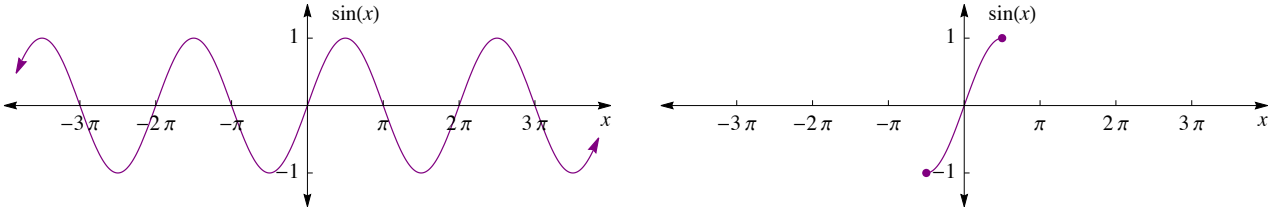
$$\sin 30° = 1/2 \quad \text{and} \quad \sin 150° = 1/2$$

If we were to put our original question in equation form, we could write: "Solve for θ: $\sin\theta = 1/2$". Solving for θ is the inverse of finding the sine of θ. In trigonometry, the inverse of the cosine function is called the **arcsine function**:

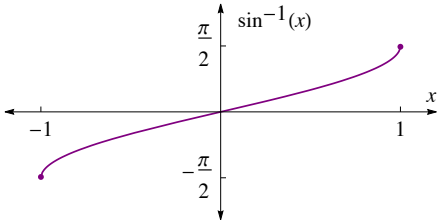
$$\theta = \arcsin(x) = \arcsin(1/2) = 60°$$

Arcsine is a function, which means that for each input, x, there can only be one output, θ. As we saw above, using the unit circle, there were two outputs: 30° and 150°. To get around this problem, the range of the arccosine function is restricted to the Quadrants I and IV.

More insight into the arcsine function by looking at the graph of the sine function:



The sine function does not pass the horizontal line test; however if we restrict the domain of $\sin(x)$ to $[-\pi/2, \pi/2]$ it does. The function $\arcsin(x)$ is the inverse of the sine function with its domain restricted. Here's it's graph:



Note that $\arcsin(x)$ is often written as $\sin^{-1}(x)$.