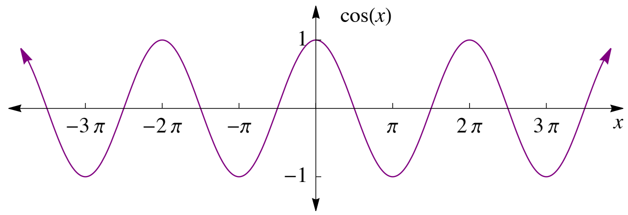
Trigonometry 29: Sin and Cos: Frequency



The period of the sine and cosine functions, $y = \sin(x)$ and $y = \cos(x)$, is 2π radians. This means that after the x-values on the graph have increased by 2π radians, the graph will have gone through one complete cycle (or oscillation or vibration). Saying this a different way:

There is one cycle every 2π radians $= \dfrac{1 \text{ cycle}}{2\pi \text{ rad}} = \dfrac{1/(2\pi) \text{ cycles}}{1 \text{ rad}} = \dfrac{1}{2\pi}$cycles per radian

For $y = \sin(2x)$, the period is π radians and there is one cycle every π radians $= \dfrac{1 \text{ cycle}}{\pi \text{ rad}} = \dfrac{1/\pi \text{ cycles}}{1 \text{ rad}} = \dfrac{1}{\pi}$ cycles per radian.

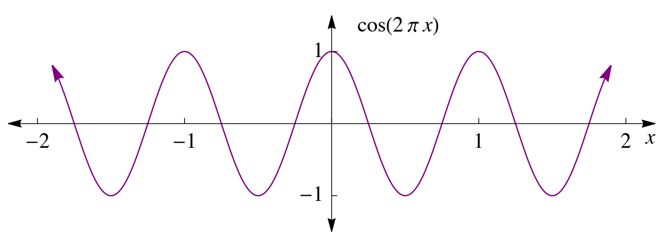
The number of cycles per radian is called the **frequency**.

b	$\cos(b\,x)$	Period [rad]	Frequency [1/rad]
1	$\cos(x)$	2π	$1/(2\pi)$
2	$\cos(2x)$	π	$1/\pi$
1/2	cos(x/2)	4π	$1/(4\pi)$
−1	$\cos(-x)$	2π	$1/(2\pi)$
π	$\cos(\pi x)$	2	1/2
$-\pi/4$	$\cos(-\pi x/4)$	8	1/8

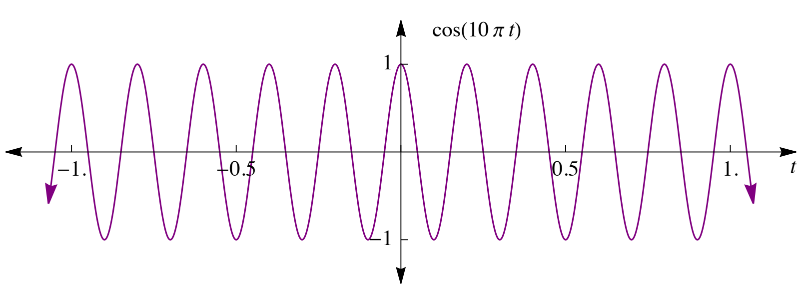
From this description of frequency and the patterns in the table, we can see that the frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \left| \frac{b}{2\pi} \right|$$

As another example, the function $y = \cos(2\pi x)$ has a frequency of $f = \left| \dfrac{b}{2\pi} \right| = \left| \dfrac{2\pi}{2\pi} \right| = 1$ cycle per radian.



In Physics, oscillations and vibrations have frequencies and those frequencies are usually measured in oscillations per second, which are Hertz (abbreviated Hz). These functions would have time on the x-axis and have a form $y = \cos(2\pi f t)$, where f is the frequency of the waveform in Hertz. For example, here is a graph of a waveform with a frequency of 5 Hz:



Quite often, especially in electrical engineering, you will see $\omega = 2\pi f$ where ω (omega) is called the **angular frequency** and it has units of radians per second. Waveforms of this type have equations that look like $f(t) = \cos(\omega t)$.