Trigonometry 30: Sin and Cos: Phase Shift

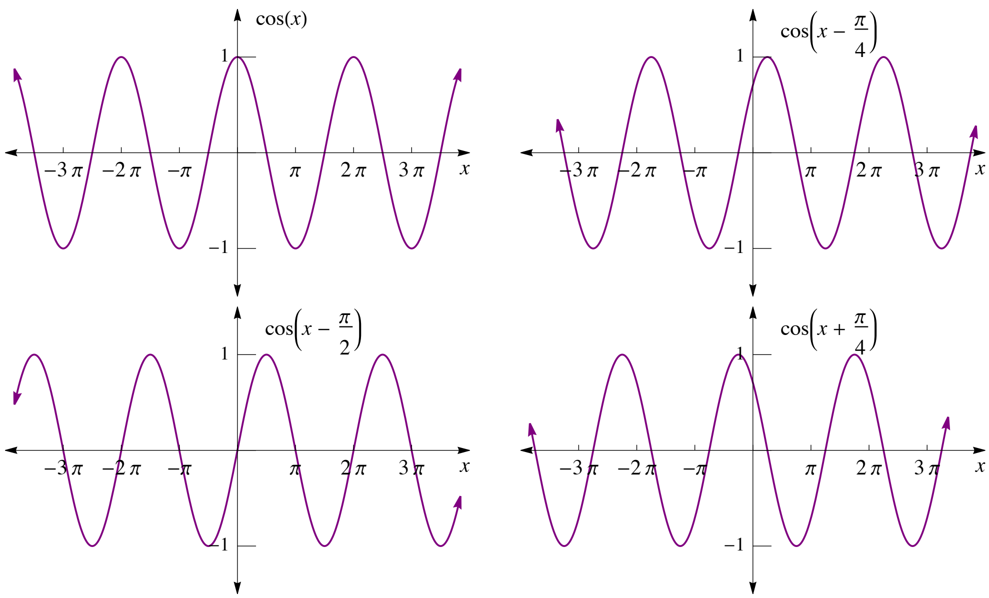
A very general way of writing the sine and cosine functions is:

$$y = a\sin(b\,x - c) + k$$
$$y = a\cos(b\,x - c) + k$$

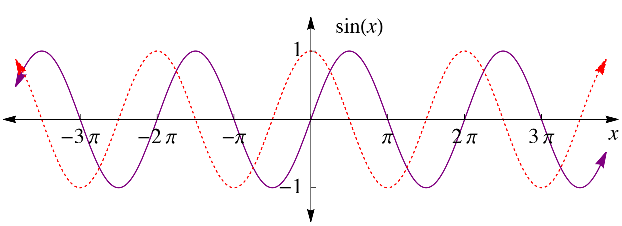
With a little algebraic manipulation, it possible to write the argument to the sine or cosine function as follows:

$$y = a\cos\left(b\left(x - \frac{c}{b}\right)\right)$$

As we have seen many times, when the independent variable in a function expression is of the form $f(x-h)$, the function $f(x)$ is translated to the right h units (if h negative the function is $f(x+h)$ and it is translated to the left). In Trigonometry, such translations are called **phase shifts**. Here are some graphs of the cosine function with phase shifts:



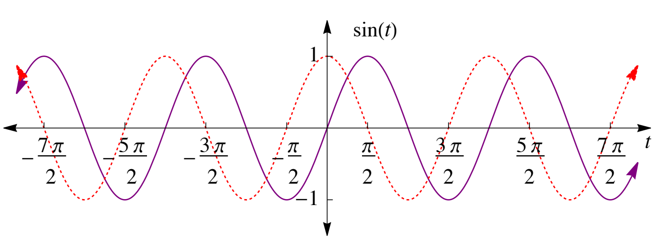
If you look closely at the graph of $\cos(x - \pi/2)$, it should look familiar. What other function behaves that way? The sine function (graphed here with the dashed, red cosine function):



This means that the following trigonometric identity, which is known as a **co-function identity**, must be true:

$$\sin(x) = \cos(x - \pi/2)$$

In other words, we say that the sine function "lags" the cosine function by $\pi/2$ radians or 90°. This concept of "lagging" may be confusing because the plot above makes it look like $\sin(x)$ is ahead of $\cos(x)$. We can resolve this dilemma by using time as the independent variable and seeing that $\cos(t)$ has a peak at 0 seconds but $\sin(t)$ does not peak until $\pi/2$ seconds later (hence the lag):



As an additional challenge, convince yourself that this co-function identity is also true: $\cos(x) = \sin(\pi/2 - x)$.