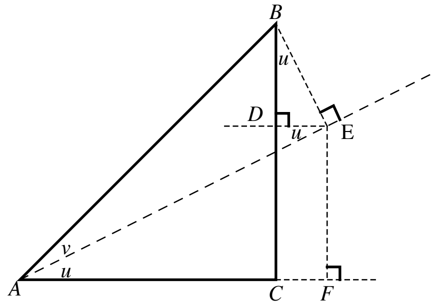
Trigonometry 37: Sum Formulas, Part II

We can also use the diagram we used to obtain the sum formula for sine to find the sum formula for cosine.



$$\cos(u+v) = \frac{A}{H} = \frac{AC}{AB} = \frac{AF - CF}{AB} = \frac{AF - DE}{AB} = \frac{AF}{AB} - \frac{DE}{AB}$$

Looking carefully, we can use the various triangles in the diagram to obtain some equalities:

Triangle	Equality
AEF	$\cos(u) = \frac{AF}{AE}$
AEB	$\cos(v) = \frac{AE}{AB}$
BDE	$\sin(u) = \frac{DE}{BE}$
AEB	$\sin(v) = \frac{BE}{AB}$

We a little manipulation, we can obtain one of the most important formulas in trigonometry, the sum formula for cosine:

$$\cos(u+v) = \frac{AF}{AB} - \frac{DE}{AB} = \frac{AF}{AB}\cdot\frac{AE}{AE} - \frac{DE}{AB}\cdot\frac{BE}{BE} = \frac{AF}{AE}\cdot\frac{AE}{AB} - \frac{DE}{BE}\cdot\frac{BE}{AB}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

Here’s an example of using the cosine sum formula:

$$\cos\left(\frac{11\pi}{12}\right) = \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4} = \frac{-1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \frac{-\sqrt{2}\left(1+\sqrt{3}\right)}{2\sqrt{2}\sqrt{2}} = -\frac{1+\sqrt{3}}{2\sqrt{2}}$$