Trigonometry 38: Difference Formulas

In a previous handout, we derived the sum formula for sine:

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

Using the parity identities, $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$, we can can manipulate this formula to obtain the **difference formula** for sine

$$\sin(u + (-v)) = \sin(u - v) = \sin(u)\cos(-v) + \cos(u)\sin(-v)$$
$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

In a previous handout, we also derived the sum formula for cosine:

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

We can also manipulate this formula to obtain the difference formula for cosine

$$\cos(u + (-v)) = \cos(u - v) = \cos(u)\cos(-v) - \sin(u)\sin(-v)$$
$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

As before, these difference formulas can be used to find more points on the unit circle:

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6} = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{2}\left(1 + \sqrt{3}\right)}{2\sqrt{2}\sqrt{2}} = \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6} = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{2}\left(\sqrt{3} - 1\right)}{2\sqrt{2}\sqrt{2}} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$