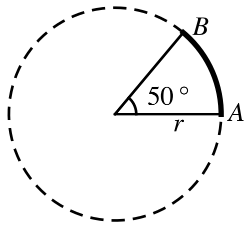
# Trigonometry Solutions #1

*Covers Circle Geometry Review*

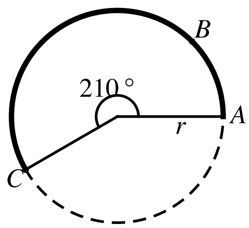
**1.** Given the length of arc AB is 1.25 cm, find $r$.




Using proportions:

$$\frac{50^{\circ}}{360^{\circ}} = \frac{1.25}{2\pi r} \implies r = \frac{360}{50} \cdot \frac{1.25}{2\pi} = \frac{9}{2\pi} \approx 1.4324\text{ cm}$$

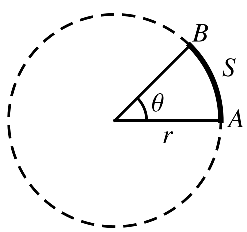
**2.** Given the length of arc ABC is 10″, find $r$.




Using proportions:

$$\frac{210^{\circ}}{360^{\circ}} = \frac{10}{2\pi r} \implies r = \frac{360}{210} \cdot \frac{10}{2\pi} = \frac{60}{7\pi} \approx 2.728\text{ ''}$$

**3.** Given the circle below, if $S$ is the length of arc AB, derive the formula for $S$.



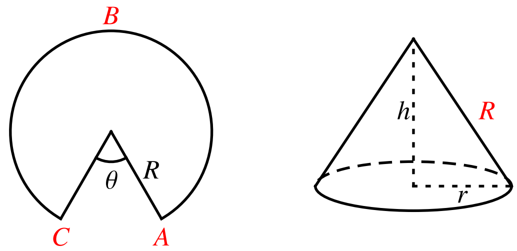

Using proportions (and degrees):

$$\frac{\theta}{360^{\circ}} = \frac{S}{2\pi r} \implies S = \frac{\theta}{360^{\circ}} 2\pi r$$

Using proportions (and radians):

$$\frac{\theta}{2\pi} = \frac{S}{2\pi r} \implies S = r\theta$$

**4.** **[Challenge]** A right circular cone is made from a piece of paper of radius $R$ by cutting out a sector with angle $\theta$ and gluing the edges together. Tip: Try it!




**a.** Find $r$, the radius of the base of the cone in terms of $R$ and $\theta$.

The circumference of the base (is the length of arc ABC):

$$C = 2\pi R - R\theta$$

For the base of the cone, $C = 2\pi r$,

$$r = \frac{C}{2\pi} = \frac{2\pi R - R\theta}{2\pi} = R - \frac{R\theta}{2\pi}$$

**b.** Find $h$, the height of the cone in terms of $R$ and $\theta$.

Using the Pythagorean Theorem:

$$h^2 = R^2 - r^2$$

$$h^2 = R^2 - \left(\frac{2\pi R - R\theta}{2\pi}\right)^2$$

$$h^2 = R^2 - \frac{4\pi^2 R^2 - 4\pi R^2\theta + R^2\theta^2}{4\pi^2}$$

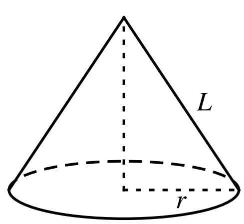
$$h^2 = \frac{4\pi^2 R^2 - 4\pi^2 R^2 + 4\pi R^2\theta - R^2\theta^2}{4\pi^2}$$

$$h^2 = \frac{4\pi R^2\theta - R^2\theta^2}{4\pi^2}$$

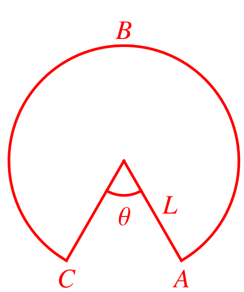
$$h^2 = \frac{R^2}{4\pi^2}\left(4\pi\theta - \theta^2\right)$$

$$h = \frac{R}{2\pi}\sqrt{4\pi\theta - \theta^2}$$

**5.** **[Challenge]** Given a right circular cone with radius $r$ and slant height $L$, show that the lateral surface area of the cone is $A = \pi r L$. Note: Lateral surface is the area of the cone that does **not** include the base.




From Question 4, the area we want is the area of a circle minus the area of a sector:



$$A = \pi L^2 - \frac{\theta}{2\pi} \pi L^2 = \pi L^2 - \frac{\theta}{2} L^2$$

Also from Question 4, part a), the relationship between $L$, $r$, and $\theta$ is:

$$r = L - \frac{L\theta}{2\pi} \quad \Longrightarrow \quad \theta = \frac{2\pi(L-r)}{L}$$

Using the above expression for $\theta$:

$$A = \pi L^2 - \frac{\theta}{2} L^2$$

$$A = \pi L^2 - \frac{L^2}{2}\left(\frac{2\pi(L-r)}{L}\right)$$

$$A = \pi L^2 - \pi L (L - r)$$

$$A = \pi L^2 - \pi L^2 + \pi r L$$

$$A = \pi r L$$