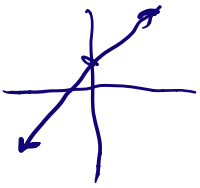
Linear Equations and Graphs (Part 2): Equation Forms

Section 1: Slope-intercept intro (KA link)

1. What is the y-intercept of $y = -6x + 4$?
For the y-intercept, the x-coordinate value is 0; the corresponding y-coordinate value is:
$y = -6(0) + 4 = 4$
y-intercept: $(0, 4)$



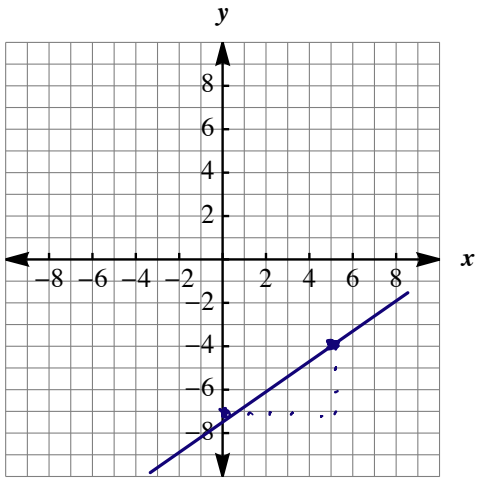
2. Write the equation of the line whose slope is -4 and the y-intercept is $(0, -5)$.
The general equation of a line in slope-intercept form is:
$y = mx + b$
where m is the slope and b is the y-coordinate of the y-intercept point:
$m = -4$ and $b = -5$
$y = -4x + (-5) = -4x - 5$

$y = (-4)x + (-5)$
$= -4x - 5$

$y = mx + b$
$m = -4$
$b = -5$

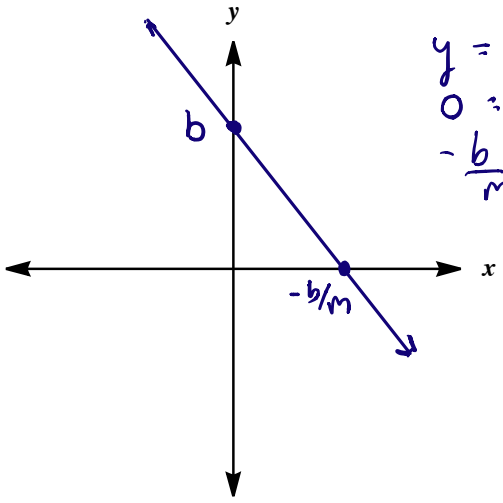
Section 2: Graph from slope-intercept form (KA link)

1. Graph $y = \frac{3}{5}x - 7$



$y = \frac{3}{5}x - 7$
$= \frac{3}{5}(5) - 7$
$= 3 - 7$
$= -4$
$(5, -4)$

2. [Challenge] Graph $y = mx + b$ where $m < 0$ and $b > 0$.



$y = mx + b$
$0 = mx + b$
$-\frac{b}{m} = \frac{mx}{m}$
$x = -b/m$

Section 3: Slope-intercept equation from graph (KA link)

1. Find the equation of the line in slope-intercept form. Use exact numbers.

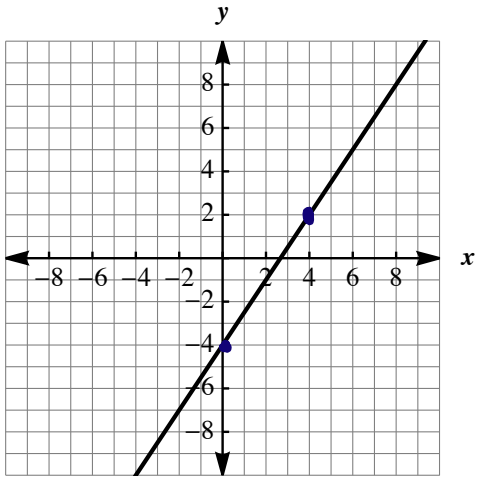
$b = -4$

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{4-0} = \frac{6}{4} = \frac{3}{2}$$

$$y = \frac{3}{2}x + (-4) = \frac{3}{2}x - 4$$

(0, -4)
(4, 2)

$y = mx + b$



Section 4: Slope-intercept from two points (KA video link)

1. Find the equation of the line with slope $-1/3$ through $(-3, 6)$ in slope-intercept form.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad y_2 - y_1 = m(x_2 - x_1)$$

$(x_2 - x_1)m = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x_2 - x_1)$

$y - y_1 = m(x - x_1)$ ←

$y - 6 = -\frac{1}{3}(x - (-3))$ = $-\frac{1}{3}(x+3)$

$m(x_2 - x_1) = y_2 - y_1$

$y_2 - y_1 = m(x_2 - x_1)$

$y = -\frac{1}{3}x - 1 + 6$

$y = -\frac{1}{3}x + 5$

$(x_1, y_1) = (-3, 6)$

$(x_2, y_2) = (x, y)$

2. [Challenge] Find the equation of the line with slope m though (x_1, y_1) in slope-intercept form.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad y_2 - y_1 = m(x_2 - x_1)$$

$y - y_1 = m(x - x_1)$ ←

$y = mx - mx_1 + y_1$

$y = mx - (mx_1 - y_1)$ or $y = mx + (y_1 - mx_1)$

$y = mx + b$

$y - y_1 = mx - mx_1$

$+y_1 \qquad +y_1$

$y = mx - mx_1 + y_1$

$= mx + (y_1 - mx_1)$

Section 5: Slope-intercept from two points (KA link)

1. Find the equation of the line through $(-10, -4)$ and $(-3, 5)$ in slope-intercept form.

$y - y_1 = m(x - x_1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - (-4) = \frac{5 - (-4)}{-3 - (-10)}(x - (-10))$

$y + 4 = \frac{9}{7}(x + 10)$

$y = \frac{9}{7}x + \frac{90}{7} - \frac{28}{7}$

$y = \frac{9}{7}x + \frac{62}{7}$

2. [Challenge] Find the equation of the line through $(-1/3, -1/2)$ and $(3/4, 4)$ in slope-intercept form.

$y - y_1 = m(x - x_1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - (-1/2) = \frac{4 - (-1/2)}{3/4 - (-1/3)}(x - (-1/3))$

$y + 1/2 = \frac{9/2}{13/12}(x + 1/3)$

$y = \frac{54}{13}x + \frac{54}{13} \cdot \frac{1}{3} - \frac{1}{2}$

$y = \frac{54}{13}x + \frac{23}{26}$

Section 6: Point-slope form (KA link)

1. Find the equation of the line through $(10, -4)$ and $(-3, -5)$ in point-slope form.

$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-4)}{-3 - 10} = \frac{-1}{-13} = \frac{1}{13}$

$y - y_1 = m(x - x_1)$ or $y - y_2 = m(x - x_2)$

$y - (-4) = \frac{1}{13}(x - 10)$ or $y - (-5) = \frac{1}{13}(x - (-3))$

$y + 4 = \frac{1}{13}(x - 10)$ or $y + 5 = \frac{1}{13}(x + 3)$

2. [Challenge] Find the equation of the line through $(1/3, -1/2)$ and $(-3/4, 4/3)$ in point-slope form.

$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4/3 - (-1/2)}{-3/4 - 1/3} = -\frac{22}{13}$

$y - y_1 = m(x - x_1)$ or $y - y_2 = m(x - x_2)$

$y - (-1/2) = -\frac{22}{13}(x - 1/3)$ or $y - 4/3 = -\frac{22}{13}(x - (-3/4))$

$y + 1/2 = -\frac{22}{13}(x - 1/3)$ or $y - 4/3 = -\frac{22}{13}(x + 3/4)$