Factors and Divisibility (Part 1—Required for Honors)

Section 1: Factors and divisibility (KA link)

1. Which monomials are divisible by $-8\,p^2\,q$ (choose all answers that apply)?

a. $16\,p^2\,q$

$$\frac{16\,p^2\,q}{-8\,p^2\,q} = -2$$

b. $72\,p^4\,q^2$

$$\frac{72\,p^4\,q^2}{-8\,p^2\,q} = -9\,p^2\,q$$

c. None of the above

2. Which monomials are factors of $-120\,a^8\,b$ (choose all answers that apply)?

a. $-50\,a^2$

$$\frac{-120\,a^8\,b}{-50\,a^2} = \frac{12}{5}\,a^6\,b$$

b. $-4\,a^5$

$$\frac{-120\,a^8\,b}{-4\,a^5} = 30\,a^3$$

c. None of the above

3. A teacher writes the following product on the board:
$(k^2-4)\,(k^2-5) = k^4 - 9\,k^2 + 20$
Karim says that $k^4 - 9\,k^2 + 20$ is a factor of $k^2 - 4$.
Marci says that $k^2 - 4$ is divisible by $k^4 - 9\,k^2 + 20$. Who is correct?

a. Only Karim
b. Only Marci
c. Both Karim and Marci
d. Neither Karim nor Marci

Section 2: Factor monomials (KA link)

1. Find the missing factor F that makes the equality true.

$$-36\,b^6 = (F)\left(9\,b^5\right)$$

$$F = \frac{-36\,b^6}{9\,b^5} = -4\,b$$

2. A rectangle has an area of $80\,p^5$ square meters and a length of $20\,p$ meters. What is the width of the rectangle?

$$A = l \cdot w \implies w = \frac{A}{l} = \frac{80\,p^5}{20\,p} = 4\,p^4$$

3. Zach factored $63\,y^8$ as $(9\,y^4)\,(7\,y^4)$
Isabella factored $63\,y^8$ as $(3\,y)\,(21\,y^7)$
Which of them factored $63\,y^8$ correctly?

a. Only Zach

b. Only Isabella

c. Both Zach and Isabella

d. Neither Zach nor Isabella

$$\left(9\,y^4\right)\left(7\,y^4\right) = 9 \cdot 7\,y^4\,y^4 = 63\,y^{4+4} = 63\,y^8$$

$$(3\,y)\left(21\,y^7\right) = 3 \cdot 21\,y\,y^7 = 63\,y^{1+7} = 63\,y^8$$

Section 3: Greatest common factor of monomials (KA link)

1. What is the greatest common factor of $24\,x^2$, $12\,x^2$ and $8\,x$?

$$24\,x^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x = 4\,x \cdot 6\,x$$

$$12\,x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x = 4\,x \cdot 3\,x$$

$$8\,x = 2 \cdot 2 \cdot 2 \cdot x = 4\,x \cdot 2$$

2. What is the greatest common factor of $32\,p^2\,q^4$ and $20\,p\,q^3$?

$$32\,p^2\,q^4 = 4\,p\,q^3 \cdot 8\,p\,q$$

$$20\,p\,q^3 = 4\,p\,q^3 \cdot 5$$

3. What is the greatest common factor of $11\,a\,b$ and $9\,a^3$?

$$11\,a\,b = a \cdot 11\,b$$

$$9\,a^3 = a \cdot 9\,a^2$$

Section 4: Factor polynomials: common factor (KA link)

1. Factor $x(x-4)-6(x-4)$ as the product of two binomials.

$(x-4)(x-6)$

2. Factor $33r^6-22r^5+88r^3$ by its greatest common monomial factor.

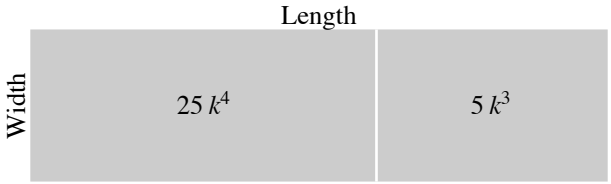
$33r^6 = 11r^3 \cdot 3r^3$

$22r^5 = 11r^3 \cdot 2r^2$

$88r^3 = 11r^3 \cdot 8$

$33r^6-22r^5+88r^3 = 11r^3 \cdot 3r^3 - 11r^3 \cdot 2r^2 + 11r^3 \cdot 8 = 11r^3\left(3r^3-2r^2+8\right)$

3. The rectangle below has an area of $25k^4+5k^3$ square meters. The width of the rectangle (in meters) is equal to the greatest monomial factor of $25k^4$ and $5k^3$. What is the length and width of the rectangle?



$25k^4 = 5k^3 \cdot 5k \implies w = 5k^3$

$A = l \cdot w \implies l = \dfrac{A}{w} = \dfrac{25k^4+5k^3}{5k^3} = \dfrac{5k^3(5k+1)}{5k^3} = 5k+1$

Section 5: Evaluate expressions using structure (KA link)

1. If $x+3y+8z=-12$, what is $24z+3x+9y$?

$24z+3x+9y = 3x+9y+24z = 3(x+3y+8z) = 3(-12) = -36$

2. If $x+y+z=6$ and $a+b=5$, what is $-8b-3z-3y-8a-3x$?

$-8b-3z-3y-8a-3x = -8a-8b-3x-3y-3z = -8(a+b)-3(x+y+z) = -8(5)-3(6) = -58$

Section 6: Warmup: factoring quadratics intro (KA link)

1. Factor $x^2+8x+15$ as the product of two binomials.

$3 \cdot 5 = 15$ and $3+5=8$

$(x+3)(x+5) = x^2+8x+15$

2. Factor $p^2 - 12p + 32$ as the product of two binomials.

$(-4)(-8) = 32$ and $-4 + (-8) = -12$

$(p - 4)(p - 8) = p^2 - 12p + 32$

3. Factor $r^2 - 3r - 18$ as the product of two binomials.

$(-6)(3) = -18$ and $-6 + 3 = -3$

$(r - 6)(r + 3) = r^2 - 3r - 18$

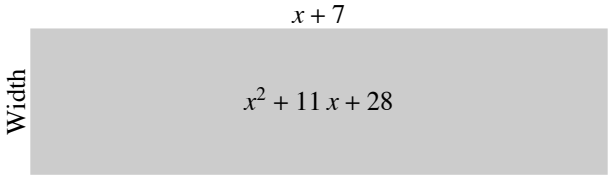
Section 7: Factoring quadratics intro (KA link)

1. Factor $x^2 - 10x + 21$ as the product of two binomials.

$(-7)(-3) = 21$ and $-7 + (-3) = -10$

$(x - 7)(x - 3) = x^2 - 10x + 21$

2. The rectangle below has an area of $x^2 + 11x + 28$ square meters and a length of $x + 7$ meters. What expression represents the width of the rectangle?



$7 + 4 = 11$ and $7 \cdot 4 = 28$

$(x + 7)(x + 4) = x^2 + 11x + 28 \implies w = x + 4$